		Monday	
9:00 - 9:45		Schrijver	
9:50 - 10:35		Luczak	
10:35 - 11:00		**Coffee**	
11:00 - 11:30	Turán	Allen	Recski
11:30 -12:00	Faudree	Han	Ruszinko
12:00 - 12:30	Simonyi	Kelly	Koshelev
12:30 - 15:00		**Lunch**	
15:00 - 15:45		Ruzsa	
15:50 - 16:35		Solymosi	
16:35 - 17:00		**Cofffee**	
17:00 - 17:30	Balogh	Chernov	Gyarmati
17:30 - 18:00	S. Wagner	Mycroft	Palmer
18:00 - 18:30	P. Wagner	Treglown	Hefetz
19:00 - 22:00		**Banquet**	

		Tuesday	
9:00 - 9:45		Wigderson	
9:50 - 10:35		Thomason	
10:35 - 11:00		**Coffee**	
11:00 - 11:30	Ferguson	Bíró	Flahive
11:30 -12:00	Johnson	Csaba	Kupavskii
12:00 - 12:30	Kokotkin	Kardos	Kochol
12:30 - 15:00		**Lunch**	
15:00 - 15:30	Skokan	Brandt	Knauer
15:30 - 16:00	Vince	Czabarka	Singhi
16:00 - 16:30	Füredi	Müller	Ellis
16:35 - 17:00		**Coffee**	
17:00 - 17:30	Bolla	Cranston	Elsasser
17:30 - 18:00	Montágh	Pluhár	Iványi
18:00 - 18:30	Person	Venkaiah	Tokushige
18:30 - 19:00	Martin	Newman	German

		Wednesday	
9:00 - 9:45		Beck	
9:50 - 10:35		Elek	
10:35 - 11:00		**Coffee**	
11:00 - 11:30	Kun	Balbuena	Bálint
11:30 -12:00	Rozovski	Peltola	Bezdek
12:00 - 12:30	Shabanov	Marx	Filimonov
12:30 - 15:00		**Lunch**	
15:00 - 15:45		Pyber	
15:50-16:35		Edmonds	
16:35 - 17:00		**Coffee**	
17:00 - 17:30	Haggkvist	Raigorodskii	Pálvölgyi
17:30 - 18:00	Sárközy	Rubanov	Sarkar
18:00 - 18:30	Gyárfás	Shitova	Szörényi
18:30 - 19:00	Lemons	Makowsky	Massow

		Thursday	
9:00 - 9:45		Thomas	
9:50 - 10:35		Shor	
10:35 - 11:00		**Coffee**	
11:00 - 11:30	Pach	Archdeacon	Borg
11:30 -12:00	Keevash	Dankelmann	Georgiou
12:00 - 12:30	Ribe-Baumann	Hubicka	Bukh
12:30 - 15:00		**Lunch**	

		Friday	
9:00 - 9:45		Blokhuis	
9:50 - 10:35		Szõnyi	
10:35 - 11:00			
11:00 - 11:30	Christofides	Blázsik	Fujita
11:30 -12:00	Doerr	Chia	Russell
12:00 - 12:30	Friedrich	Theis	Székely
12:30 - 15:00		**Lunch**	
15:00 - 15:30	Conlon	Czap	Gunderson
15:30 - 16:00	Casselgren	Jendrol	Noble
16:00 - 16:30	Cooley	Farzad	Foniok
16:35 - 17:00			
17:00 - 17:30	Gy.Y. Katona	Ait Haddadene	Moazzami
17:30 - 18:00	Lo	Griffiths	Rutherford
18:00 - 18:30	Pirzada	Zerovnik	Salazar

Invited Talks

József Beck, New Jersey (USA)

Surplus of graphs and the Lovasz local lemma

Wednesday, August 13$^{\text{th}}$, 9:00 – 9:45

Aart Blokhuis, Eindhoven (The Netherlands)

The finite field Kakeya problem

A Besicovitch set in $AG(n, q)$ is a set of points containing a line in every direction. The Kakeya problem is to determine the minimal size of such a set. We solve the Kakeya problem in the plane, and substantially improve the known bounds for $n > 4$.

Friday, August 15$^{\mathrm{th}}$, 9:00 – 9:45

Jack Edmonds, Paris (France)

Euler complexes

We present a class of instances of the existence of a second object of a specified type, in fact, of an even number of objects of a specified type, which generalizes the existence of an equilibrium for bimatrix games. The proof is an abstract generalization of the Lemke-Howson algorithm for finding an equilibrium of a bimatrix game.

Gábor Elek, Budapest (Hungary)

Testing parameters of planar graphs

Let Pl_d be the set of finite planar graphs (up to isomorphism) with vertex degree bound d. A graph parameter is just a function $f : Pl_d \to \mathbb{R}$. We call f testable if for any $\epsilon > 0$ there exists a " tester" that:

1. Picks $C(\epsilon)$ random vertices of a graph $G \in Pl_d$ then
2. takes the $C(\epsilon)$-neighborhood of the picked vertices and
3. calculate a heuristic $f^*(G)$ such that

$$Prob(|f(G) - f^*(G)| > \epsilon) < \epsilon \,.$$

We prove that the following parameters are testable.

- The independence number.
- The q-entropy if $q > d$. (the logarithm of the number of q-colorings of G divided by $|V(G)|$.)
- The edit distance from any strongly monotone graph property, e.g. 3-colorability.
- The spectral distribution at any λ.

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Tomasz Łuczak, Poznan (Poland)
Jacek Świątkowski

Random groups

In 2000 Mikhail Gromov, the founder of random group theory, wrote: *I feel, random groups altogether may grow up as healthy as random graphs, for example.* In the talk we briefly report on the current progress of this project and present some results on the evolution of random groups. This is a joint work with Jacek Świątkowski.

Monday, August 11$^{\text{th}}$, 9:50 – 10:35

László Pyber, Budapest (Hungary)

Applications of the Gowers trick

Recently Gowers proved that the group $\mathrm{PSL}(2,p)$ does not contain any product-free sets of size greater than $2p^{8/3}$. His ideas have an amazing number of applications to product-decompositions of finite groups. We describe some of these applications.

Wednesday, August 13^{th} , 15:00 – 15:45

Imre Z. Ruzsa, Budapest (Hungary)

Eine zahlentheoretische Anwendung der Graphtheorie

This is the original title of Plünnecke's 1970 paper where he invented a graph-theoretic method to study density relations between certain sumsets. Given two sets A, B in a commutative group and an integer h, we build a $(h+1)$ -partite graph with the sets A, $A+B$, ..., $A+hB$ as parts, and with edges going from each $x \in A+jB$ to all $x+b$, $b \in B$. These graphs possess a property he called "commutativity", which follows from the possibility of replacing a path $x \to x+b_1 \to x+b_1+b_2$ by $x \to x+b_2 \to x+b_1+b_2$. He established certain inequalities for the magnification properties of these graphs; the most frequently used corollary sounds as follows. If A, B are finite sets, $|A| = m$ and $|A+B| = \alpha n$, then there is a nonempty $X \subset A$ such that $|X+hB| \leq \alpha^h |X|$.

We narrate our efforts to improve and generalize this inequality, in particular, to specify this set X and to relax the requirements that the group is commutative and that the same set B is added repeatedly, which at first sight seem to form the very essence of the method.

Monday, August 11^{th} , 15:00 – 15:45

Lex Schrijver, Amsterdam (The Netherlands)

Graph invariants

Monday, August 11^{th}, 9:00 – 9:45

Peter Shor, Boston (USA)

Knots, Complexity, and Quantum Computing

It is known that evaluating an approximation to the Jones polynomial of a knot is a BQP-complete problem. That is, solving this problem is as hard, up to a polynomial factor, as any problem that can be solved on a quantum computer. We show that approximating the Jones polynomial is also complete for a weaker quantum complexity class: the one clean qubit model. This apparent contradiction is resolved by recognizing that the degree of approximation necessary for these two results is different: they are related to the complexity of representing the knots as the plat closure and the trace closure of a braid, respectively.

József Solymosi, Vancouver (Canada)

Using eigenvalues in additive combinatorics

In this talk we show examples on using graph spectral techniques in additive combinatorics. We consider three classical problems over finite fields; Sum-product bounds, Roth's theorem on 3-term arithmetic progressions, and incidence bounds between curves and points.

Tamás Szőnyi, Budapest (Hungary)

On some combinatorial problems in finite geometry

The aim of this talk is to collect some combinatorial extremum problems related to Laci Lovász' papers in finite geometry. Most of the new results are joint work with Zsuzsa Weiner.

Let us begin with some definitions and notation. The finite field with q elements ($q = p^h$, p prime) will be denoted by $\mathrm{GF}(q)$. We denote the projective (resp. affine) plane coordinatized over $\mathrm{GF}(q)$ by $\mathrm{PG}(2,q)$ (resp. $\mathrm{AG}(2,q)$).

If K is a subset of a plane then a line ℓ will be called an i-secant of K if ℓ meets K in exactly i points. Sets having no 0-secants are called blocking sets. A *blocking set* is non-trivial if it does not contain a line.

In the paper by Erdős and Lovász [1] there were some theorems related to finite geometry. The first one gives a lower bound on the number of 0-secants, if the set does not contain too many collinear points.

Result 1. (Erdős and Lovász, [1]) *A point set of size q in a projective plane of order q, with less than $\sqrt{q+1}(q+1-\sqrt{q+1})$ 0-secant lines always contains at least $q+1-\sqrt{q+1}$ points from a line.*

They also notice that by deleting some points from subplanes of order $\sqrt{q}$ (called *Baer subplanes*) one can obtain sets of q points with roughly this many 0-secants. Actually, their proof gives a slightly better result which is sharp for planes of square order having a Baer subplane: if the point set has size $q+k$ and the number of 0-secants is less than $(\sqrt{q}+1-k)(q-\sqrt{q})$, where $k \leq \sqrt{q}+1$, then the set contains at least $q-\sqrt{q}+1$ collinear points. In particular, the proof can be used to deduce Bruen's bound on the size of a non-trivial blocking set. The bound is sharp, deleting $\sqrt{q}+1-k$ points from a Baer subplane gives a set with $(\sqrt{q}+1-k)(q-\sqrt{q})$ 0-secants.

After the Erdős–Lovász result it is natural to consider stability questions for blocking sets: if a set B has at most δ 0-secants then it can be obtained from a blocking set by deleting some points (the first guess is deleting δ/q points but sometimes other bounds on the number of deleted points are more natural). Of course, it is important to find reasonable bounds on the size of the set B as well as on the number of 0-secants δ. The results were obtained with Zsuzsa Weiner using algebraic methods. So our results are valid for the plane $\mathrm{PG}(2,q)$ only, while the result of [1] is combinatorial, it is true in any projective plane of order q. From the theory of blocking sets it is natural to consider relatively small blocking sets, otherwise we do not know much about the structure of the set. The situation is quite simple if the size of the blocking set is not very small. Here we have that if $\frac{3}{2}q \leq |B| \leq 2q-1$ and $\delta \leq 2(2q-2-|B|)$, then B can be obtained from a blocking set by deleting at most $2\delta/q$ (that is at most one or two) points. Similarly, if $\frac{7}{6}q \leq |B| < \frac{3}{2}q$ and $\delta \leq q+2+3(\frac{3}{2}q-|B|)-4$, then the same conclusion holds. If $|B|$ gets smaller but it is not too close to q, then we have another bound on the number of 0-secants, namely

$$\delta < \min\left((q-1)\frac{2q+1-|B|}{2(|B|-q)}, \frac{1}{3}q\sqrt{q}\right),$$

and the conclusion is again that B is obtained by deleting at most $\frac{2\delta}{q}$ points from a blocking set.

Comparing this bound with the above mentioned more general bound of Erdős and Lovász on the number of 0-secants, this result gives something non-trivial if B has size at least $q + \frac{2}{3}\sqrt{q}$. If $|B| = q + k$, $k \geq 0$, is even smaller and $\delta \leq (\sqrt{q} + 1 - k + c)(q - \sqrt{q})$, where c is a constant (currently it is about $1/10$), then B can indeed be obtained from a Baer subplane by deleting at most $\sqrt{q} + 1 - k + c$ points and adding the correct number of points not in the Baer subplane.

However, we can allow much more 0-secants to have a stability theorem if the plane is of prime order. In this case Blokhuis proved that a blocking set with less than $\frac{3}{2}(q + 1)$ points must contain a line. Here we proved the following theorem:

Let Δ be the integer part of $\sqrt{2\varepsilon(q + 1)} - 1$. Let B be a set of points of $\mathrm{PG}(2, q)$, $q = p$ prime, that has at most $\varepsilon(q + 1)$ 0-secants for some $\varepsilon < \frac{1}{4}(q - 6)$. Suppose that $|B| < \frac{3}{2}(q + 1 - \Delta)$. Then there is a line that contains at least $q - 2\varepsilon$ points of B.In the paper of Erdős and Lovász [1] Result 1 was used to prove that if we choose $t = 4r^{3/2}\log r$ lines of a projective plane of order $r - 1 = q$ at random, then with high probability the set of chosen lines cannot be blocked by fewer than r points. It is remarked in [1] that the natural limit of the method would be $cr \log r$ and later Kahn showed this. More precisely, he proved that a set of $22q \log q$ random lines in Π_q cannot be blocked by fewer than $q + 1$ points.

Another topic related to Laci's work where there are recent developments is the direction problem for sets in an affine plane. The paper by Lovász and Schrijver [3] considers the problem of Rédei about the number of directions determined by a set of q points in $\mathrm{AG}(2, q)$, q prime. We say that a direction is *determined* by X if X contains two points spanning a line in this direction. The main result of [3] is the following.

Result 2. (Rédei–Megyesi, Lovász–Schrijver[3]) *Let p be a prime and X be a subset of the affine plane $\mathrm{AG}(2, p)$, such that $|X| = p$ and X is not a line. Then X determines at least $(p + 3)/2$ directions.*

(Lovász–Schrijver [3]) *If a p-element subset X of $\mathrm{AG}(2, p)$ determines exactly $(p + 3)/2$ directions, then in a suitable coordinate system it can be written in the form*

$$X = \{(k, k^{(p+1)/2}) : k \in \mathrm{GF}(q)\}.$$

For q not a prime, sets of q points determining at most $(q+1)/2$ directions were essentially classified by Blokhuis, Ball, Brouwer, Storme, Szőnyi as translates of vector subspaces in the affine plane $\mathrm{AG}(2, \mathrm{q})$ over a subfield of $\mathrm{GF}(q)$. There were some exceptions for the characteristic 2 and 3 cases. Recently, S. Ball found an easier proof which also handles the missing cases.

The idea of the Lovász–Schrijver proof of the theorem above was to use double power sums instead of elementary symmetric polynomials. Generalizations of this idea work nicely for planes of prime order. For sets of q points determining more than $(q + 3)/2$ directions Gács showed for q prime that the number of determined directions is at least $[2\frac{q-1}{3} + 1]$.

In Gács, Lovász, Szőnyi [2] we consider the next case, that is when q is the square of a prime:

Suppose that $q = p^2$, where p is prime and U is a set of q points in $AG(2, q)$ determining at least $\frac{q+3}{2}$ directions. Then either U is affinely equivalent to the graph of $x^{\frac{q+1}{2}}$, or the number of determined directions is at least $\frac{q+p}{2} + 1$.

References

[1] P. Erdős, L. Lovász, *Problems and results on 3-chromatic hypergraphs and some related questions*, in: Infinite and finite sets (Colloq., Keszthely, 1973; dedicated to P. Erdős on his 60th birthday), Vol. II, pp. 609–627, Colloq. Math. Soc. János Bolyai, **Vol. 10**, North-Holland, Amsterdam, 1975.

[2] A. Gács, L. Lovász, T. Szőnyi, *Directions in* $AG(2, p^2)$, to appear in Innovations in Incidence geometry

[3] L. Lovász, A. Schrijver, *Remarks on a theorem of Rédei*, Studia Scient. Math. Hungar. **16** (1981), 449–454.

Robin Thomas, Atlanta (USA)
Sergey Norin

K_t minors in large t-connected graphs

A graph G has a K_t minor if a graph isomorphic to K_t, the complete graph on t vertices, can be obtained from a subgraph of G by contracting edges. Jorgensen conjectured that every 6-connected graph with no K_6 minor has a vertex whose deletion makes the graph planar. This is of interest, because it implies Hadwiger's conjecture for graphs with no K_6 minor (which is known to be true, but Jorgensen's conjecture would give more structural information).

I conjecture that for every integer t there exists an integer N such that every t-connected graph on at least N vertices with no K_t minor has a set of at most $t - 5$ vertices whose deletion makes the graph planar. If true, this would be best possible in the sense that neither t-connectivity nor the size of the deleted set can be lowered, and for $t > 7$ some lower bound on the number of vertices is needed. Furthermore, no graph satisfying the conclusion of the conjecture has a K_t minor.

A couple of years ago we proved the conjecture for $t = 6$ in joint work with Matt DeVos, Rajneesh Hegde, Kenichi Kawarabayashi, Sergey Norin and Paul Wollan. Thus Jorgensen's conjecture holds for sufficiently big graphs. In the talk I will report on recent progress for $t > 6$ obtained in joint work with Sergey Norin.

Andrew Thomason, Cambridge (United Kingdom)
Ed Marchant

Extremal graph theory with paint

Various extensions and generalizations of the notions of extremal graph theory have been looked at down the years, some of these phrased in terms of colours, but the problems we shall discuss have not received much attention until recently. Motivation arises naturally in the context of Szemerédi's Regularity Lemma, though the actual problems, and their solutions where they exist, do not involve the lemma at all. An alternative view of the subject is that it is an extension of classical extremal graph theory to induced subgraphs.

Let H be a fixed graph whose edges are painted red and blue. Let G be a large graph, each of whose edges can be red, blue or both. We associate a weight to each edge of G; red edges have weight p, blue edges weight q, and edges of both colour have weight $p+q$. We normalize so that $p+q=1$. The extremal question is this: how large must the total edge weight of G be in order to guarantee that G contains a copy of the painted graph H?

The general problem seems difficult, and we discuss only some small special cases, in particular, only when H and G are complete graphs. In this case, the answer is known when H has at most 4 vertices (due to Richer) or when the red edges of G form a star (Richer, and also Diwan-Mubayi and Balogh-Martin). Quite precise results have been proved by Marchant for certain types of graphs (path-like or cycles). The case when the blue graph of H is $K_{3,3}$ is a particularly interesting one; a solution to this case for $1/3 \leq q \leq 2/3$ implies a solution to the edit-distance problem of Alon-Stav (first solved by Balogh and Martin). We give a simple solution for $1/3 \leq q$ but the problem becomes harder as q gets smaller. All the same, we describe a general method which pushes the frontier down as far as $1/8 \leq q$.

Avi Widgerson, Princeton (USA)

Extractors

Tuesday, August 12$^{\text{th}}$, 9:00 – 9:45

Contributed Talks

Hacène Ait Haddadene, Algiers (Algeria)
Hamadi Ahmed

Approaches that solve Combinatorial Problems for some new classes

The problems of finding the maximum clique or the optimal coloring of a graph are NP-hard in general, they can be solved in polynomial time for perfect graphs. This result is due to Grötschel et al. Unfortunately, their algorithms are based on the ellipsoid method and are, therefore, mostly of theoretical interest. It is still an open problem to find a combinatorial polynomial time algorithm to color perfect graphs or to compute the clique number of a perfect graph. However, for many classes of perfect graphs, such algorithms are known. In this paper, we present our contribution for solving these NP-Hard graph combinatorial problems for some new classes. Our algorithmic approaches based on some property of graphs are applied to k-cliques quasi-locally neighbourhood graphs denoted by $QLNC_k$ (i.e. graphs such that each induced subgraph has a vertex whose neighbourhood can be partitioned into at most k maximal cliques). We denote also the union of all these classes by $QLNC$; $QLNC = \{QLNC_k, k?\{1, 2, ...|V(G)| - 1\}\}$. Moreover, we consider the recognition problem. Chudnovsky et al. recently proved that there exists a polynomial time algorithm for recognizing perfect graphs. For several subclasses of perfect graphs such an algorithm is not yet known.

Friday, August 15$^{\text{th}}$, 17:00 – 17:30

Peter Allen, London (U.K.)
Graham Brightwell and Jozef Skokan

A connection between Ramsey number and chromatic number

We describe a new method (not involving the Regularity Lemma) for finding upper bounds for Ramsey problems. We use this method to find exactly the Ramsey number $R(P_n, H)$ for any graph H, provided n is large. We also give a sketch proof that the value does not change when we replace P_n with any connected n-vertex graph with bounded bandwidth, and that the basic structure does not change when more colours are permitted.

In particular we can find the Ramsey numbers for three or more cycles whenever one is long compared to the others.

Monday, August 11^{th}, 11:00 – 11:30

Dan Archdeacon, Burlington (USA)
Kirsten Stor

Superthrackles

We characterize those graphs that can be drawn on the plane so that every pair of edges, adjacent or non-adjacent, cross exactly once.

Thursday, August 14th , 11:00 – 11:30

Camino Balbuena, Barcelona (Spain)
E. Abajo and A. Diánez

New families of graphs without short cycles and large size

By the *extremal number* $ex(\nu; \{C_3, C_4, \ldots, C_n\})$ we denote the maximum number of edges in a graph of order v and girth at least $g \geq n+1$. The set of such graphs is denoted by $EX(\nu; \{C_3, C_4, \ldots, C_n\})$. In 1975, Erdős mentioned the problem of determining extremal numbers $ex(\nu; \{C_3, C_4\})$ in a graph of order ν and girth at least 5. In this paper, we provide some constructions of graphs of girth at least $n+1$ with large size for given values of ν. In some cases these graphs are extremal or improve known results which have been obtained using different algorithms, see [1, 2, 3].

illustrated by the examples below.

References

[1] Kwong, Y.H.H., Garnick, D.K. and F. Lazebnik, *Extremal graphs without three-cycles or four-cycles*, J. Graph Theory. **17(5)** (1993), 633–645.

[2] Lazebnik, F., Ustimenko, V.A. and A.J. Woldar, *A new series of dense graphs of high girth*, Bull. Amer. Math. Soc (N.S.). **32(1)** (1995), 73–79.

[3] Tang, J., Balbuena, C. and M. Miller, *Calculating the extremal number* $ex(\nu; \{C_3, C_4, \ldots, C_n\})$, To appear in Discrete Appl. Math.

Wednesday, August 13^{th} , 11:00 – 11:30

Vojtech Bálint, Žilina (Slovakia)
Vojtech Bálint

Survey of Packing Points in the Cube

The following problem was formulated in [1]. Let $f(n)$ be the maximal number of points, which can be packed into n–dimensional unit cube $[0,1]^n$ so, that their mutual distances are at least 1. Obviously, $f(n) = 2^n$ for $n = 1, 2, 3$. Many have shown $\log f(n) \sim \frac{1}{2}n(\log n)$. Determine $f(n)$ and sharpen the asymptotical estimate.

Theorem 1. ([2]) *It holds* $f(4) = 17$ *and the only configuration which realizes* $f(4)$ *is the set of* 16 *vertices of the unit cube* $[0,1]^4$.

Let us denote $F(n)$ any set of points, which realizes the maximal number $f(n)$ of points in the unit cube $[0,1]^n$ in mutual distances at least 1.

Theorem 2. ([2]) *If* $F(5)$ *contains all vertices of the unit cube* $[0,1]^5$, *then* $f(5) = 34$. *If* $F(6)$ *contains all vertices of the unit cube* $[0,1]^6$, *then* $f(6) = 76$.

To show an upper bound for $f(n)$ it is sufficient to construct any suitable point-set.

Lemma 3. ([3]) $f(7) \geq 184$, $f(8) \geq 481$, $f(9) \geq 994$, $f(10) \geq 2452$, $f(11) \geq 5464$ *and* $f(12) \geq 14705$.

Getting a good upper bound is usually much more difficult. In the paper [4] we proved the following upper estimates.

Theorem 4. ([4]) $f(6) \leq 192$, $f(7) \leq 576$, $f(8) \leq 2592$, $f(9) \leq 11664$, $f(10) \leq 46656$, $f(11) \leq 248832$ *and* $f(12) \leq 944784$.

The best known estimate for $f(5)$ was given in [5].

Theorem 5. ([5]) $f(5) \leq 44$.

References

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[2] Bálint, V., Bálint, V., *On the number of points at distance at least 1*, Geombinatorics 12, (2003), 157–166.

[3] Bálint, V., *Kombinatorická geometria - výber niektorých štrukturálnych problémov*, Žilina (2007).

[4] Bálint, V., Bálint, V., *On the maximum number of points at least one unit away from each other in the unit n–cube*, Periodica Mathematica Hungarica (2008), in print.

[5] Bálint, V., Bálint, V., *Ukladanie bodov do jednotkovej kocky*, Slovak Journal for Geometry and Graphics, Vol. 5 (2008), No.9, 5–12.

József Balogh, Urbana-Champaign, IL (USA)
Noga Alon, Alexandr Kostochka, Wojtek Samotij

On the Induced Subgraphs of C-Ramsey graphs

A graph is called *trivial* if it is either complete or empty. Ramsey's theorem states that every n-vertex graph contains an induced trivial subgraph of order at least $\Omega(\log n)$. We say that an n-vertex graph G is *c-Ramsey* if it does not contain a trivial induced subgraph of order greater than $c \log n$. Erdős, Faudree and Sós conjectured that every c-Ramsey graph with n vertices contains $\Omega(n^{5/2})$ induced subgraphs any two of which differ either in the number of vertices or in the number of edges, i.e., the number of distinct pairs $(|V(H)|, |E(H)|)$, as H ranges over all induced subgraphs of G, is at least $\Omega(n^{5/2})$. Recently Alon and Kostochka proved that the number of distinct pairs is at least $\Omega(n^2)$. In an ongoing work we further improve their bound.

Monday, August 11^{th} , 17:00 – 17:30

Károly Bezdek, Calgary (Canada)

On some recent progress on combinatorial properties of ball-polyhedra

The results to be discussed are centered around the following three topics: (1) characterizing edge graphs of ball-polyhedra in Euclidean 3-space; (2) rigidity of ball-polyhedra in Euclidean 3-space; (3) Reuleaux polyhedra in spherical 3-space.

Wednesday, August 13$^{\text{th}}$, 11:30 – 12:00

Péter Biró, Glasgow (UK)
Tamás Fleiner, David F. Manlove, Rob W. Irving

On the Hungarian matching schemes for secondary and higher education

Student admissions, for both secondary schools and higher education, are organised by centralised matching schemes in Hungary. In the case of secondary schools, the program, in operation since 2000, is based precisely on the original model and algorithm of Gale and Shapley [3] which appears to make it unique among similar applications. The core of the algorithm is the same for the higher education scheme, established in 1985, but this model has at least three special features that are also interesting in a theoretical sense.

The first feature, which was studied in [1], is the presence of **ties** in the system. The attempted output of the program is a so-called *stable score-limit*. It can be shown that the results of Gale and Shapley apply for this generalised model as well, namely, the applicant/college-oriented algorithms produce stable score-limits and these solutions are the best/worst possible stable score-limits for the applicants. We note that in this program the college-oriented algorithm was changed to the applicant-oriented version in 2007.

The second feature is the condition of **lower quotas**. In addition to upper quotas, here, every college may have a lower quota as well. We will show that a stable solution may not exist in this case; moreover, the problem of deciding whether a stable solution exists is NP-complete in general. We also study some relaxed versions of this problem. In our reductions we use the complexity results of Manlove *et al.* [5] and of Cornuéjols [2] (who strengthened a proof by Lovász [4]). We also present the heuristics that are used currently.

The third feature is the problem of **common quotas**. In this case, in addition to the individual quotas of the colleges, particular sets of colleges can have common quotas. Again, we show that a stable matching may not exist under such conditions that may occur in the current model and we prove that the related decision problem is NP-complete. On the other hand we show that for *nested set systems*, the problem becomes solvable by a generalised version of the Gale-Shapley algorithm. We note that this structure was present in the application until 2007, when legislative changes made the problem difficult.

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Tuesday, August 12^{th} , 11:00 – 11:30

Zoltán Blázsik, Szeged (Hungary)

Perfect d-dominating sets in de Bruijn graphs

Let n, k, d positive integers. Let $|\mathcal{A}| = n$. The de Bruijn graph is defined as:

$$B(n,k) = (V(n,k), E(n,k))$$

with $V(n,k) = \mathcal{A}^k$ as the set of vertices, and $E(n,k) = \mathcal{A}^{k+1}$ as the set of directed arcs. There is an arc from $x_1x_2\dots x_k$ to $y_1y_2\dots y_k$ if $x_2x_3\dots x_k = y_1y_2\dots y_{k-1}$.
In a graph $G = (V,E)$ a vertex y is *d-dominated* by a vertex x (or x *d-dominates* y) if there exists a directed path from x to y in G of length at most d or $x = y$. A set D of vertices is a *d-dominating set* in G if each vertex of G is d-dominated by at least one vertex of D. This set D is a *perfect d-dominating set (d-PDS)* if each vertex of G is d-dominated by exactly one vertex of D.

In the binary case M. Livingston and Q. F. Stout proved in [2] the following result (Theorem 2.12). They consider a vertex as a binary representation of an integer and refer to it by its numerical value.
For any $d \geq 1$ and for k a positive integer of the form $(d+1)m$ or $(d+1)m-1$ or $k < d$, let T_k denote a subset of the vertices of $B(2,k)$ defined as

(i) $T_1 = T_2 = \dots = T_d = \{0\}$,
(ii) $T_{(d+1)(m+1)-1} = T_{(d+1)m-1} \cup \{j : 2^{(d+1)m-1} \leq j \leq 2^{(d+1)m} - 1\}$,
(iii) $T_{(d+1)m} = T_{(d+1)m-1} \cup \{2^{(d+1))m} - 1 - s : s \in T_{(d+1)m-1}\}$.

Then the set T_k is a perfect d-dominating set for $B(2,k)$.

We proved in [1] a negative result about existence of a $2-PDS$:
In the de Bruijn graph $B(2,k)$ there is no perfect 2-dominating set if $(k-1)$ is a multiple of 3.

In this talk I would like to show constructions for $d - PDS$ and to prove negative results for other parameters n, k, d.

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Marianna Bolla, Budapest (Hungary)
Katalin Friedl, Tamás Kói, and András Krámli

Testability of the minimum balanced k-way cut density

We prove some equivalent statements for the testability of weighted graph parameters on the basis of [3], where the proof is elaborated for simple graphs.

Let G be a weighted graph on node-set $\{1,\dots,n\}$ with positive node-weights $\alpha_1,\dots,\alpha_n$ and edge-weights $\beta_{ij}\in[0,1]$. Set $\alpha_G := \sum_{i=1}^n \alpha_i$, $\alpha_V := \sum_{i\in V}\alpha_i$, and $\mathcal{G}$ is the set of all such graphs. We say that the weighted graph parameter f is testable, if for every $\epsilon>0$ there is a positive integer k such that if the node-weights of $G\in\mathcal{G}$ satisfy the condition $\max_i \alpha_i(G)/\alpha_G \le 1/k$, then $\Pr\left(|f(G)-f(\xi(k,G))|>\epsilon\right)\le\epsilon$, where $\xi(k,G)$ is a random simple graph on k nodes randomized "appropriately" from G. We show that for large n it is immaterial whether we use the graphon randomization of [2, 3] or the following ones: k nodes are chosen (with or without replacement) with probabilities α_i/α_G, and on this condition the edges come into existence independently, with probabilities of their weights.

We prove the testability of the following statistical graph parameter by means of the equivalent statements of testability. For fixed $k<n$ let $\mathcal{P}_k^*$ consist of k-partitions $P_k=(V_1,\dots,V_k)$ of the node-set such that $\alpha_{V_i}/\alpha_G\ge c$ $(i=1,\dots,k)$ with a given positive constant $c\le 1/k$. The *minimum balanced k-way cut density* of G is defined by

$$f_k(G)=\min_{P_k\in\mathcal{P}_k^*}\frac{1}{\alpha_G^2}\sum_{i=1}^{k-1}\sum_{j=i+1}^{k}E_G(V_i,V_j)\quad\text{or}\quad\min_{P_k\in\mathcal{P}_k^*}\sum_{i=1}^{k-1}\sum_{j=i+1}^{k}\left(\frac{1}{\alpha_{V_i}}+\frac{1}{\alpha_{V_j}}\right)E_G(V_i,V_j),$$

where $E_G(U,V)=\sum_{u\in U}\sum_{v\in V}\alpha_u\alpha_v\beta_{uv}$. The testability of these quantities may also follow by the theory of right-convergent sequences, ground state energies, and applications for maximum multiway cuts, see [4]. Let (G_n) be a left-convergent sequence of weighted graphs. The convergence of the adjacency spectrum of (G_n) – for simple graphs see [4] – together with the convergence of $f_k(G_n)$ may support our conjecture in [1], that $f_k(G_n)$ can be bounded from above by a function of the k smallest positive Laplacian eigenvalues of G_n, if n is large. In [1] it was bounded from below by the sum of these eigenvalues.

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Peter Borg, Malta (Malta)

Extremal t-intersecting sub-families of hereditary families

A family $\mathcal{A}$ of sets is said to be *t-intersecting* if any two sets in $\mathcal{A}$ intersect in at least t elements. If T is a t-subset of some set in a family $\mathcal{F}$, then we call the sub-family consisting of those sets in $\mathcal{F}$ which contain T a *t-star of $\mathcal{F}$*. So a t-star is trivially t-intersecting. The classical Erdős-Ko-Rado (EKR) Theorem [3] says that, if $n \geq 2r$, then the size of a 1-intersecting sub-family of $\binom{[n]}{r}$ is at most $\binom{n-1}{r-1}$, i.e. the size of a 1-star of $\binom{[n]}{r}$. Erdős, Ko and Rado [3] also showed that the largest t-intersecting sub-families of $\binom{[n]}{r}$ are the t-stars if n is sufficiently large (later Ahlswede and Khachatrian [1] remarkably obtained a characterisation of the largest t-intersecting sub-families of $\binom{[n]}{r}$ for any n, r and t).

A family $\mathcal{H}$ is said to be *hereditary* if any subset of any set in $\mathcal{H}$ is also in $\mathcal{H}$. A power set 2^X of a set X is the simplest example. Another example is a family of independent sets of a graph or matroid. We say that a set M is *$\mathcal{H}$-maximal* if M is not a subset of any set in $\mathcal{H}\backslash\{M\}$. If $\mathcal{H}$ is a hereditary family and $M_1, ..., M_k$ are the $\mathcal{H}$-maximal sets in $\mathcal{H}$, then clearly $\mathcal{H} = 2^{M_1} \cup ... \cup 2^{M_k}$; in other words, a hereditary family is a union of power sets. We denote by $\mu(\mathcal{H})$ the size of a smallest $\mathcal{H}$-maximal set in $\mathcal{H}$.

The famous Chvatal conjecture [2] claims that at least one of the largest 1-intersecting sub-families of any hereditary family is a 1-star. A simple EKR result says that this is true if $\mathcal{H}$ is $2^{[n]}$; however, for $n > t \geq 2$, the t-stars of $2^{[n]}$ are *not* the largest t-intersecting sub-families (a characterisation of the largest ones was obtained by Katona [5]), and hence the conjecture does not generalise to the t-intersection case. A generalised form of another nice conjecture, made by Holroyd and Talbot [4], is the following uniform version of Chvatal's conjecture: if $\mathcal{H}$ is hereditary and $\mu(\mathcal{H}) \geq 2r$, then at least one of the largest 1-intersecting sub-families of $\mathcal{H}^{(r)} := \{H \in \mathcal{H} \colon |H| = r\}$ is a 1-star. The EKR Theorem confirms the case $\mathcal{H} = 2^{[n]}$. The speaker recently proved the natural t-intersection generalisation of this conjecture for $\mu(\mathcal{H})$ sufficiently large, hence generalising the EKR Theorem for t-intersecting families. The talk will revolve around this result.

References

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Thursday, August 14^{th} , 11:00 – 11:30

Stephan Brandt, Ilmenau (Germany)
Jozef Miškuf, Dieter Rautenbach, and Friedrich Regen

Edge-injective and edge-surjective vertex labellings

For a graph $G = (V, E)$ we consider a k-labelling of the vertices $f : V \to \{1, 2, \dots, k\}$. Our main interest is in the induced weighting of the edges $w(uv) := f(u) + f(v)$. If this weighting is injective (surjective), we call f edge-injective (edge-surjective). The smallest (largest) k such that G has an edge-injective (edge-surjective) k-labelling is denoted by $i(G)$ ($s(G)$).

If m is the number of edges and Δ is the maximum degree of G we obtain

$$s(G) \leq \min\left\{ \left\lfloor \frac{m+1}{2} \right\rfloor, m+1-\Delta \right\} \leq \max\left\{ \left\lceil \frac{m+1}{2} \right\rceil, \Delta \right\} \leq i(G).$$

We show that in the case of trees, the first and the third inequality hold with equality by constructing an explicit labelling and indicate an application to another labelling problem that motivated our research. For complete graphs K_n with $n > 2$, the inequalities are not tight and bounds are closely related to well-studied number theoretic concepts. We show that $2m - o(m) \leq i(K_n) \leq 2m + o(m)$ based on results for Sidon sets, and conjecture that $i(G) \leq 2m$ for every graph G, though we are not able to prove any linear bound in m. We bound $s(K_n)$ based on results for additive bases. A result of Moser, Pounder, and Riddell (1969) yields $s(K_n) \leq 0.8487\frac{m}{2}$, while we derive the lower bound $s(K_n) \geq \frac{5}{9}\frac{m}{2} - o(m)$ by adapting a construction of Hämmerer and Hofmeister (1976). Experimental results suggest structural properties of the additive bases giving optimal labellings.

Boris Bukh, Princeton, NJ (USA)

Set families with a forbidden subposet

A family of subsets of $[n] = \{1, \ldots, n\}$ is naturally viewed as a subposet in the Boolean lattice $2^{[n]}$. We asymptotically determine the size of the largest family $\mathcal{F}$ of subsets of $\{1, \ldots, n\}$ not containing a given poset P if the Hasse diagram of P is a tree. This generalizes several previously known cases among which $P =$ (Sperner [4]), $P =$ (Erdős [1]), $P =$ (Katona and Tarjan [3]), and $P =$ (Griggs and Katona [2]).

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Thursday, August 14th , 12:00 – 12:30

Carl Johan Casselgren, Umeå (Sweden)
Armen S. Asratian

On path factors of $(3,4)$-biregular bigraphs

A bipartite graph is called $(3,4)$-biregular if all vertices in one part have degree three and all vertices in the other part have degree four. By a well-known conjecture of Toft, every $(3,4)$-biregular bigraph has an interval coloring (an edge coloring of a graph where the colors on the edges incident to each vertex of the graph are distinct and form an interval of integers). It was recently shown that a $(3,4)$-biregular bigraph has an interval coloring if it has a spanning subgraph whose components are paths with endpoints at 3-valent vertices and lengths in $\{2,4,6,8\}$. It was also conjectured that every simple $(3,4)$-biregular bipartite graph has such a spanning subgraph.

We present an algorithm which constructs a spanning subgraph F of a simple $(3,4)$-biregular bigraph G, such that all components of F are paths with endpoints at vertices of degree three in G. We also show that, using a variant of this algorithm, we can construct a spanning subgraph F of a simple $(3,4)$-biregular bigraph G, such that every component of F is a path of length not exceeding 22 and with endpoints at vertices of degree three in G.

Alexey Chernov, Moscow (Russia)
Andrei Raigorodskii, Kirill Mikhaylov

On Ramsey numbers for some complete distance graphs

This talk is concerned with two classical problems of extremal combinatorics.

The first one deals with *distance graphs* in the Euclidean spaces. Here by a (complete) distance graph we mean a graph $G = (V, E)$, where

$$V \subset \mathbb{R}^n, \quad E = \{\{\mathbf{x}, \mathbf{y}\} : \quad |\mathbf{x} - \mathbf{y}| = a\},$$

for a fixed positive real a. Usually, we consider only finite distance graphs, but G may be infinite as well. Distance graphs play the main role in the famous problem on finding the chromatic numbers of metric spaces (see [1], [2]).

The second problem is about *Ramsey numbers* (see [1], [3]). In the simplest case, a (diagonal) Ramsey number $R(s, s)$ is defined as the minimum natural n such that for any $G = (V, E)$ with $|V| = n$, either G or its complement to the complete graph K_n contains a complete subgraph K_s.

In this talk, we are interested in determining some analogs of Ramsey numbers for complete distance graphs. Let us assume that a sequence of distance graphs is given. For example, let $n = 4k$,

$$V_n = \{\mathbf{x} = (x_1, \ldots, x_n) : \quad x_i \in \{0, 1\}, x_1 + \cdots + x_n = 2k\},$$
$$E_n = \{\{\mathbf{x}, \mathbf{y}\} : \quad |\mathbf{x} - \mathbf{y}| = \sqrt{2k}\}, \quad G_n = (V_n, E_n), \quad N = |V_n|$$

(this sequence is motivated by the chromatic number problem, see, e.g., [2]). Then, $R(\{G_n\}; s, s)$ is the smallest integer N such that a G_n is well-defined (i.e., $|V_n| = N$) and for any spanning subgraph H of G_n, either in H or in its complement to G_n one can find an induced subgraph of G_n on s vertices.

In various situations, we obtain tight bounds for the numbers $R(\{G_n\}; s, s)$ and their generalizations.

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G.L. Chia, Kuala Lumpur (Malaysia)
Poh-Hwa Ong

On Self-Clique Graphs all of whose Cliques have Equal Size

The *clique graph* of a graph G is the graph whose vertex set is t he set of cliques of G and two vertices are adjacent if and only if the corresponding cliques have a non-empty intersection. A graph is *self-c lique* if it is isomorphic to its clique graph. In this paper, we present several results on connected self-clique graphs in which each clique has the same size k for $k = 2$ and $k = 3$.

Demetres Christofides, Birmingham (United Kingdom)
Klas Markström

Random Latin square graphs

We present new models of random graphs arising from Latin squares which were introduced in [1]. Given an $n \times n$ Latin square L with entries in $[n]$ and a random subset S of $[n]$ we obtain a random graph on $[n]$ by joining i with j if and only if either L_{ij} or L_{ji} belongs to S. These models include random Cayley graphs as a special case but are much more general.

In this talk we will only discuss some results related to the expansion properties of these graphs. The main tool used is a concentration inequality proved in [2], which can be considered as a higher-dimensional analogue of the well-known Hoeffding-Azuma inequality.

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David Conlon, Cambridge (United Kingdom)

The Ramsey multiplicity of complete graphs

In this talk we treat the following question: given a fixed t, how many monochromatic copies of K_t must one find in any two-colouring of the edges of K_n (for n large)? This is an old question of Erdős, and he proved bounds that essentially mirror the known bounds for Ramsey's theorem. In particular, for the upper bound, he showed that one has at least

$$\frac{n^t}{r(t)^t} \geq \frac{n^t}{4^{t^2}}$$

monochromatic copies of K_t.
Our main result is a large improvement on this lower bound, increasing it to

$$\frac{n^k}{C^{k^2}},$$

where $C \approx 2.18$ is an explicitly defined constant. The proof involves the construction of a recursion which we believe to be the correct analogue, for multiplicities, of the Erdős-Szekeres proof of Ramsey's theorem. The solution of this recursion is, however, markedly more complicated than that of its counterpart.

Friday, August 15^{th} , 15:00 – 15:30

Oliver Cooley, Birmingham (England)

The Loebl-Komlós-Sós conjecture for large, dense graphs

The Loebl-Komlós-Sós conjecture states that for any integers k and n, if a graph G on n vertices contains at least $n/2$ vertices of degree at least k, then G contains as subgraphs all trees with k edges ($k+1$ vertices). This is a generalisation of the famous $(n/2-n/2-n/2)$ conjecture, which covers only the case when $k = n/2$, and was recently proved for large n by Zhao [1]. Extending the method used in that paper, I will outline a proof of the Loebl-Komlós-Sós conjecture for large n, and for k linear in n. The proof makes use of the regularity lemma. The same result was also recently proved independently by Hladky and Piguet.

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Friday, August 15^{th} , 16:00 – 16:30

Daniel Cranston, Piscataway, NJ (United States)
Douglas West

Classes of 3-regular graphs that are $(7, 2)$-edge-choosable

A graph is $(7, 2)$-edge-choosable if, for every assignment of lists of size 7 to the edges, it is possible to choose two colors for each edge from its list so that no color is chosen for two incident edges. We show that every 3-edge-colorable graph is $(7, 2)$-edge-choosable and also that many non-3-edge-colorable 3-regular graphs are $(7, 2)$-edge-choosable.

A generalization of Brooks' Theorem [2] implies that every 3-regular graph is $(8, 2)$-edge-choosable. On his website, in a "Problem of the Month", Bojan Mohar [1] conjectured that every 3-regular graph is $(7, 2)$-edge-choosable. If true, this result is best possible, since it is easy to construct a 3-regular graph that is not $(6, 2)$-edge-choosable.

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Tuesday, August 12^{th} , 17:00 – 17:30

Éva Czabarka, Columbia (USA)
Peter Dankelmann, László A. Székely

The diameter of 4-colourable graphs

In 1988, Erdős, Pollack, Pach and Tuza conjectured the following:
Let $r, \delta \geq 2$ be integers and G be a connected graph with minimum degree δ.

1. If G is K_{2r}-free and $(r-1)(3r-2)|\delta$, then $\operatorname{diam}(G) \leq \frac{(r-1)(3r-2)n}{(2r^2-1)\delta} + O(1)$
2. If G is K_{2r+1}-free and $(3r-1)|\delta$, then $\operatorname{diam}(G) \leq \frac{(3r-1)n}{r\delta} + O(1)$

They constructed graphs that show that the upper bounds are best possible. We consider a weakened version of this conjecture, where we replace the condition K_{m+1}-free by m-colourable, and make the first step towards proving the weakened conjecture by showing that for any $\delta \geq 2$, the diameter of a 4-colourable graph G is at ost $\frac{5n}{2\delta} - 1$.

Július Czap, Košice (Slovakia)
Stanislav Jendroľ

Parity vertex colourings of plane graphs with face constrains

Consider a vertex colouring of a connected plane graph G. A colour c is used k times by a face α of G if it appears k times along the facial walk of α. Two natural problems arise.

1. A vertex colouring φ is a *weak parity vertex colouring* (wpv colouring) of a connected plane graph G with respect to its faces if each face of G uses at least one colour an odd number of times. Problem is to determine the minimum number $\chi_w(G)$ of colours used in a wpv colouring of G.

2. A vertex colouring φ is a *strong parity vertex colouring* (spv colouring) of a 2-connected plane graph G with respect to the faces of G if for each face α of G and each colour c of φ, no vertex or an odd number of vertices incident with α are coloured by c. Problem is to find the minimum number $\chi_s(G)$ of colours used in an spv colouring of G.

We have proved that $\chi_w(G) \leq 4$ for every connected plane graph with minimum face degree at least 3.

We present our other recent results and open questions concerning the above mentioned problems.

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Béla Csaba, Bowling Green (USA)
Jeff Abrahamson and Ali Shokoufandeh

Optimal Random Matchings on Trees and Applications

We consider tight upper- and lower-bounds on the expected total length of the optimal matching between two random point sets distributed among the leaves of a hierarchically separated tree. Specifically, given two point sets $R = \{r_1, ..., r_n\}$ and $B = \{b_1, ..., b_n\}$ distributed uniformly and randomly on the m leaves of a λ-Hierarchically Separated Tree with branching factor b such that each one of its leaves are of depth δ, we show that the expected total length of the optimal matching between R and B is $\Theta(\sqrt{nb}\sum_{k=1}^{h}(\sqrt{b}\lambda)^k)$, for $h = \min(\delta, \log_b n)$. This technique allows us to provide bounds on the expected total length on other metric spaces via approximate embeddings into hierarchically separated trees. In particular, we reproduce the results concerning the expected optimal transportation cost in $[0, 1]^d$ (except for $d = 2$) and prove upper bounds on finite approximations of self-similar sets, e.g., the Cantor set, and various fractals.

Peter Dankelmann, Durban (South Africa)
Ivan Gutman, Simon Mukwembi and Henda Swart

The Edge-Wiener Index of Graphs

The Wiener index of a connected finite graph is defined as the sum of the distances between all pairs of vertices. It has been studied in several papers and under different names, for example total distance, transmission, average distance or mean distance. In this talk we introduce an edge-analogue to the Wiener index: the edge-Wiener index $W_e(G)$ of a connected finite graph G is defined as the sum of the distances between all pairs of edges in a connected graph, where the distance between two edges is defined as the distance between the vertices representing them in the line graph of G.

We give bounds on W_e in terms of order and size. In particular we prove the asymptotically sharp upper bound $W_e(G) \leq \frac{2^5}{5^5} n^5 + O(n^{9/2})$ for graphs of order n.

Benjamin Doerr, Saarbrücken (Germany)
Tobias Friedrich, Anna Huber, Thomas Sauerwald

Quasirandom Rumor Spreading

Motivated by Jim Propp's quasirandom model of random walks (cf. e.g. [2]), we propose and analyse a quasirandom analogue of the classical "randomized rumor spreading" problem (also known as push model for disseminating information in networks).

In the randomized rumor spreading model, we start with one node of a graph knowing a "rumor". Then in each round each node that knows the rumor chooses a neighbor at random and informs it of the rumor. Results of Frieze and Grimmett [3] show that in a complete graph on n vertices, this simple protocol succeeds in spreading the rumor from one node to all others within $(\log_2(n) + \ln(n))(1 + o(1))$ rounds. For the network being a hypercube or a random graph $G(n, p)$ with $p \geq (1 + \varepsilon)(\log n)/n$, again $\mathcal{O}(\log n)$ rounds suffice, see Feige, Peleg, Raghavan, and Upfal [4].

In the quasirandom model, we assume that each node has a (cyclic) list of its neighbors. Once informed, it starts at a random position of the list, but from then on informs its neighbors in the order of the list. Surprisingly, irrespective of the orders of the lists, the above mentioned bounds still hold. In addition, we also show an $\mathcal{O}(\log n)$ bound for sparsely connected random graphs $G(n, p)$ with $p = (\log n + f(n))/n$, where $f(n) \to \infty$ and $f(n) = \mathcal{O}(\log \log n)$. Here, the classical model needs $\Theta(\log^2(n))$ rounds.

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[3] A. Frieze and G. Grimmett, *The shortest-path problem for graphs with random arc-lengths*, Discrete Applied Mathematics **10** (1985), 57–77.

David Ellis, Cambridge (UK)

Intersecting Families of Permutations and the Cameron Ku conjecture

A family of permutations $\mathcal{A} \subset S_n$ is said to be *intersecting* if any two permutations in $\mathcal{A}$ agree at some point, i.e. for any $\sigma, \pi \in \mathcal{A}$, $\exists i \in [n]$ such that $\sigma(i) = \pi(i)$. Deza and Frankl [1] showed that for such a family, $|\mathcal{A}| \leq (n-1)!$. Cameron and Ku [2] showed that if equality holds then $\mathcal{A} = \{\sigma \in S_n : \sigma(i) = j\}$ for some $i, j \in [n]$. They conjectured a 'stability' version of this result, namely that there exists a constant $c > 0$ such that any intersecting family $\mathcal{A} \subset S_n$ of size at least $(1-c)(n-1)!$ is contained within $\{\sigma \in S_n : \sigma(i) = j\}$ for some $i, j \in [n]$ (we call such a family 'centred'). They also made the stronger 'Hilton-Milner' type conjecture that for $n \geq 6$, if $\mathcal{A} \subset S_n$ is a non-centred intersecting family, then $\mathcal{A}$ cannot be larger than the family $\{\sigma \in S_n : \sigma(1) = 1, \sigma(i) = i \text{ for some } i > 2\} \cup \{(12)\}$, which has size $(1 - 1/e + o(1))(n-1)!$, and that the extremal families are precisely the double cosets of this family.

We will sketch a proof the stability conjecture, and also the Hilton-Milner type conjecture for n sufficiently large. One of our key tools will be an extremal result for cross-intersecting families of permutations: we prove that for $n \geq 4$, a cross-intersecting pair of families of permutations $\mathcal{A}, \mathcal{B} \subset S_n$ satisfies $|\mathcal{A}||\mathcal{B}| \leq ((n-1)!)^2$, with equality iff $\mathcal{A} = \mathcal{B} = \{\sigma \in S_n : \sigma(i) = j\}$ for some $i, j \in [n]$. This was conjectured by Leader [3].

Tuesday, August 12$^{\text{th}}$, 16:00 – 16:30

Robert Elsässer, Paderborn (Germany)
Thomas Sauerwald

On Bounding the Cover Time

In this talk, we study the relationship between the cover time of a graph and the runtime of randomized broadcast defined by Feige et al. [4]. The cover time of a graph is the expected number of time steps required by a random walk to visit all vertices of the graph. Randomized broadcast spreads a rumor, known initially by exactly one node, to all nodes by letting in each time step every *informed* node forward the rumor to a neighbor selected independently and uniformly at random.

We provide a fairly tight characterization of graph classes for which the cover time and broadcast time capture each other. In particular, we strongly confirm for these graph classes the intuition formulated by Chandra et al. [3] that "the cover time is an appropriate metric for the performance of certain kinds of randomized broadcast algorithms". By using new probabilistic and combinatorial techniques, we prove the following main results.

- For any graph G of size n we have $R(G) = O(\frac{|E|}{\delta} \log n)$, where $R(G)$ denotes the quotient of the cover time and broadcast time and δ is the minimum degree of G. This result leads to new combinatorial inequalities relating eigenvalues to some kind of edge-expansion, which might be of independent interest.
- For any d-regular (or almost d-regular) graph G it holds that $R(G) = \Omega(\frac{d^2}{n} \cdot \frac{1}{\log n})$. Together, with our upper bound on $R(G)$, this lower bound strongly confirms the intuition of Chandra et al. for *all* graphs with minimum degree $\Theta(n)$.
- Conversly, for any d we construct d-regular graphs for which $R(G) = O(\max\{\sqrt{n}, d\} \cdot \log^2 n)$. Since for any expander it holds that $R(G) = \Theta(n)$, the strong relationship given above does not always hold if d is polynomially smaller than n.

Our results show that the relationship between cover time and randomized broadcast is substantially stronger than the relationship between any of these two and the mixing time of the corresponding random walk (or the related spectral gap or conductance, cf. [1, 2]).

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Tuesday, August 12^{th} , 17:00 – 17:30

Babak Farzad, St. Catharines (Canada)

Vizing's Conjecture for Planar Graphs of Maximum Degree 5

Vizing's List Chromatic Index Conjecture states that every simple graph G is $(\Delta(G)+1)$-edge-choosable where $\Delta(G)$ is the maximum degree of G. The conjecture is proved for simple graphs with $\Delta \leq 4$ [2] and for simple planar graphs with $\Delta \geq 9$ [1]. The case $\Delta = 5$ seemed to be hard; e.g., the conjecture was proved for planar graphs without 4-cycles and $\Delta \neq 5$ [4], or for those without intersecting 3-cycles and $\Delta \neq 5$ [3]. We prove Vizing's conjecture for planar graphs of maximum degree 5.

References

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Ralph Faudree, Memphis, Tennessee (USA)

Saturation Numbers

A graph G is an H-saturated graph if G does not contain H as a subgraph, but $G \cup \{e\}$ contains a copy of H for any edge e not in G. The *saturation number* of H, denoted by $sat(H, n)$, is the minimum number of edges in an H-saturated graph G of order n. A survey of some of the classical results on saturation numbers will be presented, also with a comparison of the saturation number $sat(H, n)$ with the Turán extremal number $ex(H, n)$. However, the focus will be on some recent results on saturation numbers. This will include saturation numbers for disjoint union of complete graphs, generalized fans, books and generalized books, and special classes of trees.

David Ferguson, London (UK)

Three colour Ramsey numbers for cycles.

Denote by $R(G_1, G_2, G_3)$ the minimum integer N such that any 3-colouring of the edges of the complete graph K_N contains a monochromatic copy of the G_i coloured with colour i for some $i = 1, 2, 3$.

Bondy and Erdős [1] conjectured that $R(C_n, C_n, C_n) = 4n - 3$ for every odd $n > 3$. This was confirmed for large values of n by Kohayakawa et al. [4] who built upon earlier fundamental work of Łuczak [2].

In [3] Figaj and Łuczak found the asymptotic value of the Ramsey number for a triple of long cycles of mixed parity. Here we build on their work to find the exact value of this Ramsey number for large n:

Defining $\ll x \gg$ to be the largest even integer not greater than x and $< x >$ to be the largest positive odd integer not greater than x, the following holds: For $\alpha_1, \alpha_2, \alpha_3 > 0$ there exists n_0 such that for $n \geq n_0$

i. for $\alpha_1 \geq \alpha_2$,

$$R(C_{\ll \alpha_1 n \gg}, C_{\ll \alpha_2 n \gg}, C_{<\alpha_3 n>}) = \max \quad \{ \quad 2 \ll \alpha_1 n \gg + \ll \alpha_2 n \gg -3, \\ 0.5 \ll \alpha_1 n \gg + 0.5 \ll \alpha_2 n \gg + < \alpha_3 n > -2\};$$

ii. for $\alpha_2 \geq \alpha_3$,

$$R(C_{\ll \alpha_1 n \gg}, C_{<\alpha_2 n>}, C_{<\alpha_3 n>}) = \max \quad \{ \quad 4 \ll \alpha_1 n \gg -3, \ll \alpha_1 n \gg +2 < \alpha_2 n > -3\}.$$

References

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Vladislav Filimonov, Moscow (Russia)

Covering plane sets

In the paper, some problems are studied that are concerned with the classical Borsuk problem on dividing sets in the Euclidean space into parts of smaller diameter as well as with the well-known Nelson – Hadwiger problem on the chromatic number of the Euclidean space.

New estimates are obtained for the values $d_n = \sup d_n(\Phi)$ and $d'_n = \sup d'_n(\Phi)$, where suprema are taken over all the sets of diameter 1 in the plane and the quantities $d_n(\Phi)$, $d'_n(\Phi)$ are defined, for a given $\Phi \subset \mathbb{R}^2$, as follows:

$$d_n(\Phi) = \inf \left\{ x \in \mathbb{R}^+ : \Phi \subseteq \Phi_1 \cup \cdots \cup \Phi_n, \forall i \ \operatorname{diam} \Phi_i \leqslant x \right\},$$

$$d'_n(\Phi) = \inf \left\{ x \in \mathbb{R}^+ : \Phi \subseteq \Phi_1 \cup \cdots \cup \Phi_n, \ \forall i \ \forall X, Y \in \Phi_i \ XY \neq x \right\}.$$

In other words, we are dealt with covering sets in the plane either by sets of definitely many times smaller diameter or by sets without pairs of points which are at a given distance apart.

The sequence d_n has already been investigated by Lenz, Borsuk, and Grünbaum. However, in our paper, substantially better methods are used in order to improve a number of previously known results.

At the same time, the problem of finding the elements of the sequence d'_n is proposed here for the first time.

Previous results are given in [1], [2].

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Mary Flahive, Corvallis (USA)

Balancing R-ary Gray codes

An R-ary Gray code is an ordering of all R^n n-strings from the alphabet $\{0, 1, \ldots, R-1\}$ with the property that any two consecutive n-strings differ in exactly one coordinate with difference ± 1. This generalizes the Binary Reflected Gray Code designed by Frank Gray in the 1950s to facilitate the relay of information through many repeaters. An important distinguishing characteristic among Gray codes is the relative uniformity of transition spectrum, the set of counts of digit-changes in the code. We give new constructions of R-ary Gray codes whose transition spectrum is close-to-uniform.

Tuesday, August 12^{th} , 11:00 – 11:30

Jan Foniok, Zurich (Switzerland)
Claude Tardif

Generating tractable CSP by means of adjoint functors

A family $\mathcal{T}$ of digraphs is a *complete set of obstructions* for a digraph H if for an arbitrary digraph G the existence of a homomorphism from G to H is equivalent to the non-existence of a homomorphism from any member of $\mathcal{T}$ to G. A digraph H is said to have *tree duality* if there exists a complete set of obstructions $\mathcal{T}$ consisting of orientations of trees. We show that if H has tree duality, then its arc graph δH also has tree duality, and we derive a family of tree obstructions for δH from the obstructions for H.

Furthermore we generalise our result to right adjoint functors on categories of relational structures. We show that these functors always preserve tree duality, as well as polynomial CSPs and the existence of near-unanimity functions.

Tobias Friedrich, Saarbrücken (Germany)
Joshua Cooper, Benjamin Doerr, Joel Spencer

Deterministic Random Walks on Regular Trees

Jim Propp's rotor router model is a deterministic analogue of a random walk on a graph. Instead of distributing chips randomly, each vertex serves its neighbors in a fixed order.

Cooper and Spencer [2] show a remarkable similarity of both models. If an (almost) arbitrary population of chips is placed on the vertices of a grid Z^d and does a simultaneous walk in the Propp model, then at all times and on each vertex, the number of chips deviates from the expected number the random walk would have gotten there, by at most a constant. This constant is independent of the starting configuration and the order in which each vertex serves its neighbors. The constant is known precisely for $d \leq 2$ [1, 3].

These results raise the question if all graphs do have this property. With quite some effort, we are now able to answer this question negatively. For the graph being an infinite k-ary tree ($k \geq 3$), we show that for any deviation D there is an initial configuration of chips such that after running the Propp model for a certain time there is a vertex with at least D more chips than expected in the random walk model. However, to achieve a deviation of D it is necessary that at least $\exp(\Omega(D^2))$ vertices contribute by being occupied by a number of chips not divisible by k in a certain time interval.

References

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Shinya Fujita, Gunma National College of Technology, Maebashi (Japan)
Ken-ichi Kawarabayashi

Some recent results on non-separating subgraphs in highly connected graphs

All graphs considered here are finite, undirected, and without loops or multiple edges. We report some results on non-separating subgraphs in highly connected graphs. In [5], Thomassen proved that any triangle-free k-connected graph has a contractible edge. Starting with this result, there are several known results concerning the existence of contractible elements in k-connected graphs which do not contain specified subgraphs. In particular, Kawarabayashi [4] proved that any k-connected graph without K_4^- subgraphs contains either a contractible edge or a contractible triangle. Motivated by these results, we proceed to research and obtained the following results:

Theorem 1. *Let k be an integer with $k \geq 6$. If G is a k-connected graph such that G does not contain $D_1 = K_1 + (K_2 \cup P_3)$ as a subgraph and G does not contain $D_2 = K_2 + (k-2)K_1$ as an induced subgraph, then G has either a contractible edge which is not contained in any triangle or a contractible triangle. (Here, P_3 means a path of length 2.)*

Theorem 2.*Let k be an integer with $k \geq 2$. Suppose G is a k-connected graph with minimum degree at least $\lfloor 3k/2 \rfloor + 2$. Then G has an edge e such that $G - V(e)$ is still k-connected.*

Theorem 3.*Let k be an integer with $k \geq 2$. If G is k-connected, then G contains either C_4 or a connected subgraph of order 3 whose contraction results in a k-connected graph. (Here, C_4 means a quadrilateral.)*

In this talk, we will mention about the details of the above three results and also we will further report some other latest results.

References

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Friday, August 15$^{\text{th}}$, 11:00 – 11:30

Zoltán Füredi, Budapest (Hungary)
Lale Özkahya

Generalized Turan problems
Looking for even-cycles in the hypercube

Given graphs Q and P the *generalized Turan number* $\mathrm{ex}(Q, P)$ denotes the maximum number of edges of a P-free subgraph of Q. We consider the case when P is the cycle of lenght $2k$ and Q^n is the hypercube, (i.e., Q^n is n-regular and it has 2^n vertices). Erdős conjectured that

$$\mathrm{ex}(Q^n, C_4) = (\frac{1}{2} + o(1))e(Q^n) \quad (?)$$

Fan Chung showed an upper bound 0.623 and that
$\mathrm{ex}(Q^n, C_6) \geq (1/4)e(Q^n)$, moreover that $\mathrm{ex}(Q^n, C_{4k}) = o(e(Q^n))$.
There are further results concerning C_{10} by Alon et al., by Axenovich et al., by A. Thomason et al., and more. Here we show that

$$\lim_{n\to\infty} \mathrm{ex}(Q^n, C_{2k})/e(Q^n) = 0.$$

for all C_{2k}, except for C_4, C_6, and possibly for C_{10}. This is a joint work with Lale Özkahya.

Nicholas Georgiou, Bristol (U.K.)
Małgorzata Kuchta, Michał Morayne, and Jarosław Niemiec

The best-choice problem for partially ordered sets

The classical best-choice or "best secretary" problem is defined as follows. A player is told that n elements form a total order, and (s)he wishes to choose the maximal element. However, the elements are revealed one at a time in a random order and after each element is revealed, the player must decide whether or not to select this element, using only the order information given by the revealed elements. (If all the elements are revealed then the player must select the last one.) The problem is to find a strategy that maximises the probability of selecting the maximal element. This well known problem has an optimal strategy that achieves a success probability of $1/e$ (asymptotically, as $n \to \infty$).

We consider a variant of this problem, where the n elements are ordered partially (not totally) and this order is unknown to the player. (The number n is still known to the player). The elements are revealed in the same manner, with the same conditions of selection on the player. The player succeeds if (s)he selects any maximal element of the partial order. Here the problem is to find the optimal *universal* strategy, i.e., a strategy achieving the maximum δ such that the probability of success is at least δ for *any* partial order. Preater proposed a universal strategy which he proved is successful with probability at least $1/8$. We show that the obvious improvement to this strategy (also due to Preater) achieves success with probability at least $1/4$. We also show that this strategy can do no better than this: there are partial orders for which the probability of success is at most $1/4 + \varepsilon$.

Oleg German, Moscow (Russia)
Nikolay Moshchevitin

On linear forms of a given diophantine type

The talk is devoted to our joint result with Moshchevitin concerning Diophantine approximations for linear forms, similar to the following result concerning simultaneous best approximations obtained in [1]:

Theorem (Akhunzhanov and Moshchevitin, 2006). *For each positive integer m there are explicit positive constants A_m, B_m with the following property. Let $\psi(p) : \mathbb{R}_+ \to \mathbb{R}_+$ be an arbitrary non-increasing function and let $\psi(1) \leq A_m$. Then there is an uncountable set of vectors $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_m) \in \mathbb{R}^m$, such that for all $p \in \mathbb{Z}_+$*

$$\max_{1\leq i\leq m} ||p\alpha_i|| \geq \frac{\psi(p)}{p^{1/m}} \left(1 - B_m\psi(p)\right),$$

but the inequality

$$\max_{1\leq i\leq m} ||p\alpha_i|| \leq \frac{\psi(p)}{p^{1/m}} \left(1 + B_m\psi(p)\right)$$

has infinitely many solutions in positive integers p.

Our result concerns the "dual" problem, which is approximating zero with the values of a linear form at integer points. We consider the best approximations for linear forms and require them to be of a given order defined by a non-increasing sequence $\{\psi_k\}$. However, the restriction on *all* the best approximations effects in the weaker exponent in the remainder. For reasons of simplicity we give our result in the three-dimensional case:

Theorem (German and Moshchevitin, 2008). *There are explicit positive constants A, B with the following property. Let $\{\psi_k\}_{k=1}^{\infty}$ be an arbitrary non-increasing sequence of positive real numbers, $\psi_1 < A$. Then there is an uncountable set of vectors $\boldsymbol{\alpha} = (\alpha_1, \alpha_2) \in \mathbb{R}^2$, such that all the best approximations $\mathbf{m}_k$ for the linear form $L_{\boldsymbol{\alpha}}$ (we assume that $L_{\boldsymbol{\alpha}}(\mathbf{x})$ equals the inner product of $\boldsymbol{\alpha}$ and $\mathbf{x}$ for every $\mathbf{x} \in \mathbb{R}^2$) satisfy the condition*

$$\psi_k - B\psi_k^{5/3} < \|L_{\boldsymbol{\alpha}}(\mathbf{m}_k)\| \cdot |\mathbf{m}_k|^2 \leq \psi_k + \frac{1}{2}\psi_k^{5/3}.$$

References

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Simon Griffiths, Cambridge (UK)

One-way subgraphs in oriented graphs

A one-way subgraph of an oriented graph is a set of edges $E(A, B)$ for sets of vertices A and B for which $e(B, A) = 0$. We write $ow(G)$ for the size of the largest one-way subgraph of G. We discuss best possible lower bounds on $ow(G)$ for the class of regular oriented graphs and the class of oriented graphs without isolated vertices, these lower bounds being $\Omega(n)$ and $\Omega(n/logn)$ respectively. We shall give an idea of the proofs and of the examples that show these results are best possible, both the proofs and the examples are found using the probabilistic method.

David Gunderson, Winnipeg (Canada)
George Grätzer

On pseudocomplemented meet semilattices

Let M be a finite pseudocomplemented meet semilattice. By Glivenko's theorem, every nontrivial interval $[0, a]$ in M is pseudocomplemented, and the set $S(a)$ of all pseudocomplements in $[0, a]$ forms a boolean lattice B_i. We describe all sequences $\langle b_1, b_2, \ldots, b_n \rangle$ of integers, for which there exists a finite pseudocomplemented meet semilattice M so that for each i, $b_i = |\{a \in M : S(a) \cong B_i\}|$, and there is no $a \in M$ with $S(a) \cong B_{n+1}$. Furthermore, for each such sequence, M can be taken to be a lattice. This result settles a problem raised by the first author in 1971.

Friday, August 15th , 15:00 – 15:30

András Gyárfás, Budapest (Hungary)
Gábor N. Sárközy and Stanley Selkow

Ramsey type results in G(allai)-colorings

A G-coloring of a complete graph is an edge coloring that does not contain triangles colored with three different colors. Since G-colorings generalize 2-colorings, it is natural to study how Ramsey type results for 2-colorings carry over to G-colorings. A basic tool for that is the following theorem, discovered by many authors in different forms and contexts, perhaps first (implicitly) in Gallai's work ([1]) on comparability graphs. The form below is from [2].

Any G-coloring can be obtained by substituting G-colored complete graphs into vertices of a nontrivial 2*-colored complete graph.*

Based on this structure theorem, certain results - every 2-colored complete graph has a monochromatic spanning tree; has a monochromatic spanning diameter three subgraph - carry over almost automatically to G-colorings. In case of some other results more work is needed, usually to work out a a weighted version of the statement to be carried over to G-colorings. It may also happen that a result for 2-colorings has no counterpart for G-colorings. For example, for $n \geq 6$, K_n has a monochromatic triangle in every 2-coloring but no K_n has this property for every G-coloring.

The phenomenon showed in the last example disappears if the number of colors is fixed in G-colorings. Let $R(k)$ be the smallest integer m such that there is a monochromatic triangle in every k-coloring of K_m. Let $GR(k)$ be defined similarly, restricting ourselves to G-colorings with k colors. It is a very difficult open problem to narrow the known bounds of $R(k)$ ($c^k \leq R(k) \leq [ek!] + 1$). What about $GR(k)$?

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Katalin Gyarmati, Budapest (Hungary)
Máté Matolcsi and Imre Ruzsa

Sums of sets of integers

For finite sets of integers $A_1, A_2 \ldots A_n$ we study the cardinality of the n-fold sumset $A_1 + \cdots + A_n$ compared to those of $n-1$-fold sumsets $A_1 + \cdots + A_{i-1} + A_{i+1} + \ldots A_n$. We prove a superadditivity and a submultiplicativity property for these quantities. We also examine the case when the addition of elements is restricted to an addition graph between the sets.

Monday, August 11$^{\text{th}}$, 17:00 – 17:30

Roland Häggkvist, Umeå (SWEDEN)

Odd facts about $(3,4)$-biregular bigraphs

An (a,b)-biregular bigraph (or an (a,b)-graph) is a bipartite graph where the vertices in one part have degree a and all vertices in the other part have degree b.
Among other things there shall be shown that

- every 2-edge-connected $(3,4)$-graph of girth 6 has a $P_{4,3}$-decomposition, where a $P_{k,k-1}$ is a path of length $2k-2$ with k vertices in the first part and $k-1$ vertices in the second part,
- there exists an infinite number of 2-edge-connected $(3,4)$-graphs where every $\{P_{2,1}, P_{3,2}, \dots\}$-factor is a $\{P_{3,2}, P_{5,4}\}$-factor,
- a 2-edge-connected $(3,4)$-graph on $7k$ vertices contains a 2-regular subgraph H on $6k$ vertices.

Wednesday, August 13$^{\text{th}}$, 17:00 – 17:30

Hiệp Hàn, Berlin (Germany)
Mathias Schacht

Loose Hamiltonian Cycles In Uniform Hypergraphs With Large Minimum Degree

Dirac's Theorem guarantees the existence of a Hamiltonian cycle in a graph provided its minimum degree is at least $n/2$. As a generalisation, we say that a cycle in a k-uniform hypergraph $\mathcal{H}$ is l-Hamiltonian if it covers all vertices and every two consecutive edges intersect in exactly l vertices.

In this talk we prove an approximate Dirac type theorem for loose Hamiltonian cycles, i.e. when $l < k/2$. More precisely, we show that for all integers k, $l < k/2$, and for every real $\gamma > 0$ there is an n_0 such that for all $n > n_0$ the following holds: Every k-uniform hypergraph $\mathcal{H}$ on n vertices whose minimum $(k-1)$-degree is at least $\left(\frac{1}{2(k-l)} + \gamma\right) n$ contains a l-Hamiltonian cycle. This result is best possible up to the error term γ.

Monday, August 11$^{\text{th}}$, 11:30 – 12:00

Dan Hefetz, Zurich (Switzerland)
Huong T. T. Tran and Annina Saluz

An application of the Combinatorial NullStellenSatz to a graph labelling problem

An *antimagic labelling* of a graph G with m edges and n vertices, is a bijection from the set of edges of G to the set of integers $\{1, \ldots, m\}$, such that all n vertex sums are pairwise distinct, where a vertex sum is the sum of labels of all edges incident with that vertex. A graph is called *antimagic* if it admits an antimagic labelling. In [2], Ringel has conjectured that every simple connected graph, other than K_2, is antimagic. In this work, we prove a special case of this conjecture. Namely, we prove that if G is a graph on $n = p^k$ vertices, where p is an odd prime and k is a positive integer, that admits a C_p-factor, then it is antimagic. The case $p = 3$ was proved in [3]. Our main tool is the Combinatorial NullStellenSatz (c.f. [1]).

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Jan Hubicka, Prague (Czech Republic)
Jaroslav Nešetřil

Universal structures as shadows of ultrahomogeneous structures

Countable graph U is said to be universal for a family of countable graphs $\mathcal{F}$, if $U \in \mathcal{F}$ and if every graph $G' \in \mathcal{F}$ is an induced subgraph of U. It is a long lasting problem to characterize classes $\mathcal{F}$ containing universal graph U. Cherlin et al. characterized many of such classes using the algebraic closure. In special case this proves that all classes of graphs determined by forbidding homomorphisms from a finite set of graphs always contain universal graph. We show a new and more explicit proof of this result using amalgamation argument similar to earlier proofs for classes of graphs with givcen odd girth. The more constructive proof has relations to duality theorems for graph homomorphisms and ultrahomogeneous metric spaces (Urysohn metric space). (This is ajoint work with Jaroslav Nešetřil)

Thursday, August 14$^{\text{th}}$, 12:00 – 12:30

Antal Iványi, Budapest (Hungary)

Latin and Sudoku algorithms

Let n be a positive integer, $N = \{0, 1, \ldots, n\}$ and $N^+ = \{1, 2, \ldots, n\}$ be alphabets. A **Latin square** L of order n is an $n \times n$ sized array, in which each row and column contains the elements of N^+ exactly once [2]. A **Latin puzzle** of order n is an $n \times n$ sized array containing the elements of N.

If $n = m^2$, then an $n \times n$ sized array can be divided into $m \times m$ disjunct subarrays (called **blocks**) of size $m \times m$. Let $M = \{0, 1, 2, \ldots, m^2\}$, $M^+ = \{1, 2, \ldots, m^2\}$. A **sudoku square** S of order m is an $m^2 \times m^2$ sized array, in which each row, column and block contains the elements of M^+ exactly once. A **sudoku** puzzle of order m is an $m^2 \times m^2$ sized array containing the elements of M [1, 3, 4].

Let A_0, A_1, A_2, ... be solving algorithms [5] of Latin and Sudoku puzzles, and let R_i be the union of the algorithms A_0, A_1, ... A_i. A **uniqueness set** $U_i(L)$, resp. $U_i(S)$ ($i = 0, 1, 2, \ldots$) is such subarray of L, resp. S, which has exactly one solution, and is solvable by R_i. An **irreducible uniqueness set** $I_i(L)$, resp. $I_i(S)$ ($i = 0, 1, 2, \ldots$) is such uniqueness set, which without its any element has more solutions.

We present results and problems on the complexity of different sudoku versions, further on the spectrum of the sizes of irreducible uniqueness sets of Latin and sudoku squares for algorithms R_i (where A_0 = Baby_step, A_1 = Naked_single, A_2 = Hidden_single, ...).

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Stanislav Jendroľ, Košice (Slovakia)
Kristína Budajová and Stanislav Krajči

Parity vertex colouring of graphs

A parity path in a vertex colouring of a graph is a path along which each colour is used an even number of times. Let $\chi_p(G)$ be the least number of colours in a vertex colouring of G having no parity path. It is proved that for any graph G there is

$$\chi(G) \leq \chi_p(G) \leq |V(G)| - \alpha(G) + 1$$

where $\chi(G)$ and $\alpha(G)$ is the chromatic number and the independence number of G, respectively. The bounds are tight. This result is improved for trees. Namely, if T is a tree with diameter $diam(T)$ and radius $rad(T)$, then

$$\left\lceil \log_2 \left(2 + diam(T)\right)\right\rceil \leq \chi_p(T) \leq 1 + rad(T)\,.$$

The bounds are tight.

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Robert Johnson, London (UK)
John Talbot

Vertex Turán Problems in the Hypercube

The discrete hypercube Q_n is the graph with vertex set $\{0,1\}^n$ and two vertices being adjacent if they differ in precisely one coordinate.

What is the maximum size of a subset of $V(Q_n)$ which induces a Q_2-free graph? This question was answered by Kostochka who proved that the largest such subset contains $\lceil \frac{2}{3} 2^n \rceil$ vertices (in fact the unique largest such subset is obtained by deleting every third layer from the cube). The analagous question for subsets of Q_n inducing Q_d-free graphs was was posed by Alon, Krech and Szabo but for $d > 2$ little is known.

We consider a more general extremal question for the hypercube. Given $F \subset V(Q_d)$, there is a natural notion of being F-free. Specifically $S \subset V(Q_n)$ is F-free if there is no embedding $i : Q_d \to Q_n$ with $i(F) \subset S$. The problem of determining the largest F-free set is very natural but does not seem to have been addressed previously in this generality.

We solve this problem asymptotically in a number of natural cases. In particular we generalise of Kostochka's result for $F = Q_2$ and prove a local stability result for the structure of near-extremal sets. We also consider the effect of forbidding a family of subgraphs and exhibit a non-principality result analogous to that shown for k-graphs by Balogh and refined by Mubayi and Pikhurko.

Finally, we pose some questions and make some conjectures towards a more complete theory of such vertex Turán problems in the hypercube.

František Kardoš, Košice (Slovakia)
Daniel Kráľ, Jozef Miškuf and Jean-Sébastien Sereni

Perfect matchings in fullerene graphs

A fullerene graph is a planar cubic 3-connected graph with only pentagonal and hexagonal faces. We show that fullerene graphs have exponentially many perfect matchings.

Gyula Y. Katona, Budapest (Hungary)

Hamiltonian Chains in Hypergraphs

A hamiltonian chain in an r-uniform hypergraph is a cyclic ordering of its vertices, such that every consecutive r-tuple forms an edge of the hypergraph. For $r = 2$ this is an ordinary hamiltonian cycle in a graph. Since there are many interesting questions about hamiltonian cycles in graphs, we can try to answer these questions for hypergraphs, too.

We have several results concerning the following questions:

1. What is the best bound in a Dirac type theorem?

2. A hypergraph is hamiltonian if it contains a hamiltonian-chain and it is k-edge-hamiltonian if by the removal of any k edges a hamiltonian hypergraph is obtained. What is the minimum number of edges in a k-edge-hamiltonian, r-uniform hypergraph on n vertices?

3. What is the maximum number of edges in an r-uniform hypergraph on n vertices which has no hamiltonian chain?

Peter Keevash, London (UK)

A hypergraph blowup lemma

We obtain a hypergraph generalisation of the graph blow-up lemma proved by Komlós, Sarközy and Szemerédi, showing that quasirandom hypergraphs with no atypical vertices behave like complete partite hypergraphs for the purpose of embedding bounded degree subhypergraphs. In the course of our arguments we also obtain various useful lemmas concerning hypergraph regularity that have independent interest, including a characterisation in terms of the frequency of certain subcomplexes. There are many potential applications of our theorem to hypergraph generalisations of results for graphs that were obtained with the blow-up lemma. We illustrate the method with a hypergraph generalisation of a result of Kühn and Osthus on packing bipartite graphs.

Thursday, August 14th , 11:30 – 12:00

Luke Kelly, Birmingham (United Kingdom)
Deryk Osthus and Daniela Kühn

Cycles of Given Length in Oriented Graphs

The most famous case of the Caccetta-Häggkvist conjecture states that any oriented graph G with minimum outdegree $\delta^+(G) \geq \lceil |G|/3 \rceil$ contains a (directed) triangle. I will discuss a generalisation of this question asking what outdegree forces a cycle of length exactly ℓ in an oriented graph, and provide a complete answer for $\ell \geq 4$, $\ell \not\equiv 0$ modulo 3. I will discuss the perhaps surprising result that for any $\epsilon > 0$ there exists n_0, ℓ such that if G is an oriented graph on $n \geq n_0$ vertices and $\delta^+(G), \delta^-(G) \geq \epsilon n$ then G contains a cycle of length ℓ. I will also discuss related results on Hamilton cycles and pancyclicity in oriented graphs.

Kolja Knauer, Berlin (Germany)
Stefan Felsner

Distributive Polytopes

A D-polytope is a polytope which is closed under componentwise maximization and minimization. This is, the point set of a D-polytope forms a distributive lattice in the dominance order on the Euclidean space. We characterize D-polytopes in terms of their bounding halfspaces. Examples are given by order polytopes or more generally by the "polytropes" of Joswig and Kulas [2].

Besides being a nice combination of geometrical and order theoretical concepts, D-polytopes are a unifying generalization of several distributive lattices arising from graphs. In fact every D-polytope corresponds to a directed graph with edge parameters, such that every point in the polytope corresponds to a vertex potential of the graph. Alternatively an edge-based description of the point set can be given, which is dual to flows with gains and losses.

These models specialize to distributive lattices that have been found on flows of planar graphs by Khuller, Naor and Klein [3], α-orientations of planar graphs by Felsner [1], and c-orientations of graphs by Propp [4].

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Tuesday, August 12th , 15:00 – 15:30

Martin Kochol, Bratislava (Slovakia)

Solution of the Grünbaum's conjecture

By a classical result of Tait [1], the Four Color Theorem is equivalent with the statement that each 2-edge-connected 3-regular planar graph has a 3-edge-coloring. An embedding of a graph in a surface is called polyhedral if its dual has no multiple edges and loops. A conjecture of Grünbaum [2], presented in 1968, states that each 3-regular graph with a polyhedral embedding in an orientable surface has a 3-edge-coloring. With respect to the result of Tait, it aims to generalize the four color theorem for any orientable surface. We present a negative solution of this conjecture, showing that for each orientable surface of genus at least 5, there exists a 3-regular non 3-edge- colorable graph with a polyhedral embedding in the surface.

ed by the examples below.

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Andrei Kokotkin, Moscow (Russia)
Andrei Raigorodskii

On large subgraphs of distance graphs having small chromatic number

The chromatic number $\chi(\mathbb{R}^d)$ of the space $\mathbb{R}^d$ is defined as the smallest quantity of colours one should use in order to paint $\mathbb{R}^d$ so that among points of the same colour, one would not find a pair of points at the unit distance apart (see [2], [1]).

Obviously, $\chi(\mathbb{R}^1) = 2$. However, the problem of determining the value of $\chi(\mathbb{R}^2)$ is surprisingly hard. We still know only that $4 \leq \chi(\mathbb{R}^2) \leq 7$.

One of important interpretations of the just-mentioned problem may be done in terms of graph theory. Indeed, it is easily seen that $\chi(\mathbb{R}^2)$ is exactly equal to the ordinary chromatic number of the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ whose set of vertices $\mathcal{V}$ coincides with $\mathbb{R}^2$ and whose set of edges $\mathcal{E}$ consists of all the pairs of points in $\mathcal{V}$ such that the distance between them is 1. Any subgraph of the graph $\mathcal{G}$ is called *distance graph*.

In the 50's P. Erdős and N.G. de Bruijn proved, in particular, that there exists a finite distance graph G such that $\chi(G) = \chi(\mathbb{R}^2)$ (see [3]). This result is of course based on the Axiom of Choice.

On the other hand, there are different reasons to believe that $\chi(\mathbb{R}^2) = 4$. Nevertheless, no one knows how to prove the bound $\chi(G) \leq 4$ for *any* (finite) distance graph in the plane.

The main result of this presentation is in

Theorem. *In any distance graph* $G = (V, E)$ *in the plane, one can find an induced subgraph on more than* $0.91|V|$ *vertices whose chromatic number does not exceed 4.*

A series of similar results is obtained. Moreover, the results are used to estimate the threshold for the property "a random graph in the Erdős – Rényi model can be realized as a distance graph in the plane" (see [1]).

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Vitaliy Koshelev, Moscow (Russia)
Andrei Raigorodskii

On the Erdős – Szekeres problem

In this talk, we would like to discuss one of the most famous problems of combinatorial geometry and Ramsey theory.

The problem was proposed in 1935 by P. Erdős and G. Szekeres (see [1]). It consists in determining, for any $n \geq 3$, the smallest number $g(n)$ such that in every set of $g(n)$ points in $\mathbb{R}^2$ in general position, one can find n vertices of a convex n-gon.

In our work, we are especially interested in an important modification of the above-mentioned problem, which is due to Erdős, too (see [2]). Namely, we consider the quantity $h(n)$ whose definition differs from that of the value $g(n)$ by transforming the expression "a convex n-gon" into "a convex *empty* n-gon".

It is probably rather surprising that the properties of the values $g(n)$ and $h(n)$ are quite different. While $g(n)$ is well-defined for each $n \geq 3$, the quantity $h(n)$ does not exist for $n \geq 7$ (see [3]).

The most intriguing and non-trivial situation was that of $n = 6$. Till 2006, the question whether $h(6)$ does exist has been remaining open. T. Gerken was the first who showed that $h(6) < \infty$. Moreover, he obtained the explicit bound $h(6) \leq g(9) \leq 1717$ (see [4]).

In 2007 we succeeded in improving Gerken's result, and our estimate was $h(6) \leq \max\{g(8), 400\}$ (see [5]). It is known that $g(8) \leq 463$ (see [3]), so we actually got the bound $h(6) \leq 463$. At the same time, Erdős and Szekeres conjectured that $g(n) = 2^{n-2} + 1$ (see [1], [3]). This means that one should have $h(6) \leq \max\{65, 400\}$. In this case, the "parasitic" number 400 plays a very bad role in our estimate. Now, we can remove it.

In the talk, we will present a survey of various results concerning $g(n)$ and $h(n)$, and we will also give several ideas of how to prove the estimate $h(6) \leq g(8)$.

The work is done under the support of the grant 06-01-00383 of the RFBR, of the grant MD-5414.2008.1 of the Russian President, by the grant NSh-691.2008.1 of the Leading Scientific Schools of Russia, and by the grant of "Dynastia" foundation.

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Gábor Kun, Simon Fraser University (Canada)

An asymptotic version of the Bollobás-Catllin-Eldridge conjecture

We say that the graphs G and H with n vertices pack if the graphs can be embedded to the same vertex set with no overlapping edges. Bollobás, Eldridge and indepenedently Catlin conjectured that if $\Delta(G)+1)(\Delta(H)+1) \leq n+1$ holds for the maximal degrees then G and H pack. We prove an asymptotic version of the conjecture:

For every $\varepsilon > 0$ there is D such that $\Delta(G), \Delta(H) > D$ and $\Delta(G)\Delta(H) < (1-\varepsilon)n$ imply that G and H pack.

Wednesday, August 13^{th} , 11:00 – 11:30

Andrei Kupavskii, Moscow (Russia)
Andrei Raigorodskii

On dividing three-dimensional sets into five parts of smaller diameter

This work is concerned with the classical Borsuk partition problem (see, e.g., [1], [2], [2]). More precisely, let k, n be natural numbers, and assume that Φ is an arbitrary bounded non-singleton point set in $\mathbb{R}^n$. Putting $\operatorname{diam}\Phi = \sup\limits_{X,Y\in\Phi} \rho(X,Y)$, where $\rho(X,Y)$ is the standard Euclidean distance, we define functions $d_k^n(\Phi)$, d_k^n as follows:

$$d_k^n(\Phi) = \inf\{x \geq 0 : \ \Phi = \Phi_1 \cup \Phi_2 \cup \ldots \cup \Phi_k, \ \operatorname{diam}\Phi_i \leq x\}, \quad d_k^n = \sup_{\Phi, \ \operatorname{diam}\Phi=1} d_k^n(\Phi).$$

The problem of determining the quantities d_k^n is well-studied in the cases $n \leq 2$. However, the case of $n = 3$ is already much more complicated.

First of all, it is readily seen that $d_1^3 = d_2^3 = d_3^3 = 1$. Another, more sophisticated, old result is given by the inequality $d_4^3 \geq \sqrt{\left(3+\sqrt{3}\right)/6}$.

D. Gale conjectured in 1953 that the last inequality is tight. However, it is still unknown whether Gale's conjecture is true or false. The best upper bound here is $d_4^3 \leq 0.98$, which is due to V.V. Makeev and L. Evdokimov.

Of course the problem of finding the value d_5^3 is even harder than its analog for d_4^3. In 1982 M. Lassak showed that $d_5^3 \leq \sqrt{\left(35+\sqrt{73}\right)/48} = 0.9524...$

The main result we want to present here consists in improving Lassak's estimate. More precisely, we prove the following

Theorem. *The inequality holds* $d_5^3 \leq 0.9425$.

The result is based on a refined construction of a universal covering system in $\mathbb{R}^3$.

The work is done under the financial support of the grant 06-01-00383 of the Russian Foundation for Basic Research, of the grant MD-5414.2008.1 of the Russian President, by the grant NSh-691.2008.1 of the Leading Scientific Schools of Russia, and by the grant of "Dynastia" foundation.

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Nathan Lemons, Budapest (Hungary)
Győri, Ervin

Hypergraphs avoiding cycles of a given length

We give upper bounds for both uniform and non-uniform hypergraphs avoiding cycles of a given length. We use the loosest, most general definition of a cycle which is commonly associated with Berge. We provide constructions which show our bounds to be sharp up to the constant factor for some small cases. If the order of magnitude of the extremal bipartite graphs containing no cycle of length $2k$ is the same as that of extremal bipartite graphs containing no cycles of length less than or equal to $2k$, then our constructions are sharp (up to the constant factor) for all positive integers.

Wednesday, August 13$^{\text{th}}$, 18:30 – 19:00

Allan Lo, Cambridge (United Kingdom)

Cliques in Regular Graphs

For a simple graph G, let $k_r(G)$ denote the number of r-cliques in G. What is the minimum $k_r(G)$ in graphs with n vertices and e edges? The best general bound so far is due to Bollobás. For $r = 3$, there are results due to Lovász and Simonovits, and Fisher for $e \leq \frac{2}{3}\binom{n}{2}$ and recently, Razborov proved an asymptotically sharp bound for all e. Nikiforov has proved an asymptotically sharp bound for all e and $r \leq 4$.

We consider the case when G is regular. Unlike the general case, for n odd and $e \geq \frac{1}{5}n^2$, $k_3(G) > 0$. We give an exact lower bound for n odd and e just below $\frac{1}{2}\binom{n}{2}$. Also, we investigate the behaviour of $k_3(G)$ asymptotically for $e \geq \frac{1}{2}\binom{n}{2}$.

Friday, August 15$^{\text{th}}$, 17:30 – 18:00

Johann A. Makowsky, Haifa (Israel)
Benjamin Godlin and Tomer Kotek

Evaluation of graph polynomials

A graph polynomial $p(G, \bar{X})$ can code numeric information about the underlying graph G in various ways: as its degree, as one of its specific coefficients or as evaluations at specific points $\bar{X} = \bar{x}_0$. In this paper we study the question how to prove that a given graph parameter, say $\omega(G)$, the size of the maximal clique of G, cannot be a fixed coefficient or the evaluation at any point of the Tutte polynomial, the interlace polynomial, or any graph polynomial of some infinite family of graph polynomials.

Our result is very general. We give a sufficient condition in terms of the connection matrix of graph parameter $f(G)$ which implies that it cannot be the evaluation of any graph polynomial which is invariantly definable in $CMSOL$, the Monadic Second Order Logic augmented with modular counting quantifiers. This criterion covers most of the graph polynomials known from the literature.

Wednesday, August 13^{th} , 18:30 – 19:00

Ryan Martin, Ames, Iowa (USA)
József Balogh

On the edit distance function for graphs

Given a hereditary property, $\mathcal{H}$, the *edit distance* of a graph G from $\mathcal{H}$ is the minimum number of edge-additions and edge-additions required to transform G into a member of $\mathcal{H}$ and is denoted $\mathrm{Dist}(G, \mathcal{H})$. The *edit distance function* is

$$f_{\mathcal{H}}(p) := \lim_{n\to\infty} \frac{1}{\binom{n}{2}} \max\left\{\mathrm{Dist}(G,\mathcal{H}) : |V(G)| = n, |E(G)| = p\binom{n}{2}\right\}.$$

For any hereditary property, $\mathcal{H}$, $f_{\mathcal{H}}(p)$ is both continuous and concave. The quantity of interest is the maximum value of f. We give examples of hereditary properties for which this maximum can occur at p^*, for any rational $p^* \in [0,1]$ as well as one for which $p^* = \sqrt{2}-1$. In the process, we develop a weighted generalization of Turán's theorem, which may be of independent interest.

This function has been studied in [2] and by Alon and Stav (see, e.g. [1]). It uses ideas developed by previous authors, for example Prömel and Steger [4] and Bollobás and Thomason [3].

This is an active area of research and, time permitting, we will describe some new results.

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Dániel Marx, Budapest (Hungary)
Martin Grohe

On tree width, bramble size, and expansion

A *bramble* in a graph G is a family of connected subgraphs of G such that any two of these subgraphs have a nonempty intersection or are joined by an edge. The *order* of a bramble is the least number of vertices required to cover every subgraph in the bramble. Seymour and Thomas proved that the maximum order of a bramble in a graph is precisely the tree width of the graph plus one. We prove that every graph of tree width at least k has a bramble of order $\Omega(k^{1/2}/\log^2 k)$ and size polynomial in n and k, and that for every k there is a graph G of tree width $\Omega(k)$ such that every bramble of G of order $k^{1/2+\epsilon}$ has size exponential in n. To prove the lower bound, we establish a close connection between linear tree width and vertex expansion. For the upper bound, we use the connections between tree width, separators, and concurrent flows.

Mareike Massow, Berlin (Germany)
Stefan Felsner, Graham Brightwell

Diametral Pairs of Linear Extensions

Given a finite poset P, we consider pairs of linear extensions of P with maximal distance. The distance of two linear extensions L_1, L_2 is the number of pairs of elements of P appearing in different orders in L_1 and L_2. A diametral pair maximizes the distance among all pairs of linear extensions of P.

Deciding if P has two linear extensions of distance at least k is NP-complete for general P, and can be solved in polynomial time for posets of width 3.

In [1], Felsner and Reuter conjectured that in every diametral pair at least one of the two linear extensions reverses a critical pair of P. We give a counterexample disproving this conjecture. On the other hand, we show that the conjecture holds for almost all posets.

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Balázs Montágh, London (United Kingdom)

New lower bounds on Zarankiewicz and Turán numbers

Let $z(n, s, t)$ be the smallest integer k such that every $(0, 1)$ matrix of size $n \times n$ with k 1s must have a set of r rows and s columns such that the corresponding $r \times s$ submatrix is made up only of 1s. By the seminal paper of Kővári, Sós and Turán,

$$z(n, s, t) < (t-1)^{1/s} n^{2-1/s} + \frac{s-1}{2} n$$

That is,

$$\log_n \left(z(n, s, t) - (t-1)^{1/s} n^{2-1/s} \right) \leq 1.$$

On the other hand, Wilson proved that

$$\log_n \left(z(n, 2, t) - (t-1)^{1/2} n^{3/2} \right) \geq 1/2$$

for infinitely many n.

We shall halve the gap of the second exponent, proving

$$\log_n \left(z(n, 2, t) - (t-1)^{1/2} n^{3/2} \right) \geq 3/4$$

for infinitely many n. The proof uses quasifields, providing a rare example in which, apparently, quasifields lead to better results in extremal graph theory than fields do. However, in the particular case of $t = 3$, fields provide a similar result on a stronger question: giving a new lower bound on the Turán number $K_{2,3}$.

Tobias Müller, Eindhoven (The Netherlands)
Robert J. Waters

Circular choosability is rational

The circular choosability of a graph is a list-version of the circular chromatic number that was introduced by Mohar in 2002 and has since been studied by a number of authors. One of the nice properties that the circular chromatic number enjoys is that it is a rational number for all finite graphs. A fundamental question posed by Zhu is whether the same holds for the circular choosability. In my talk I give a sketch of the proof that this is indeed the case and I will mention some other known results and open problems concerning circular choosability.

Richard Mycroft, Birmingham (UK)
Peter Keevash, Daniela Kühn, Deryk Osthus

Hamilton Cycles in Hypergraphs

A well-known theorem of Dirac states that a graph on n vertices with minimum degree $n/2$ contains a Hamilton cycle. A natural question to look at is whether analogues of this result can be found for k-graphs.

This is complicated by there being more than one sensible definition of a Hamilton cycle in a hypergraph, sharing the properties that consecutive edges intersect and that every vertex is included in the cycle. Rödl, Rucinski and Szemerédi examined *tight* Hamilton cycles, in which consecutive edges intersect in $k-1$ vertices, and showed that, asymptotically, the minimum degree threshold to guarantee such a cycle is $n/2$, just as in the graph case. We instead investigated *loose* Hamilton cycles, in which consecutive edges intersect in a single vertex. Kühn and Osthus previously showed that for $k=3$, the minimum degree threshold to guarantee the existence of a loose Hamilton cycle is, asymptotically, $n/4$. Using the recent hypergraph blow-up lemma by Keevash, we were able to find the analogous result for any k, showing that the minimum degree threshold is, asymptotically, $\frac{n}{2k-2}$.

Monday, August 11$^{\text{th}}$, 17:30 – 18:00

Mike Newman, Wellington and Ottawa (New Zealand and Canada)
Dillon Mayhew, Geoff Whittle

Excluded Minors for Real-Representability

The well-known Rota's conjecture asserts that for a finite field, the matroids represented over that field can be characterized by a finite set of excluded minors.

In this talk we show that for an infinite field K, every K-representable matroid appears as a minor of an excluded minor for K-representability. This answers a cojecture of Jim Geelen. Our proof is constructive, and has consequences for finite fields also.

Tuesday, August 12^{th} , 18:30 – 19:00

Steven Noble, Brunel University (United Kingdom)
Bill Jackson, Dave Wagner

An Inequality for the Tutte Polynomial

Let G be a graph without loops or bridges and $T_G(x,y)$ be its Tutte polynomial. A conjecture of Merino and Welsh states that $\max\{T_G(2,0), T_G(0,2)\} \geq T_G(1,1)$. (Here $T_G(2,0)$, $T_G(0,2)$ and $T_G(1,1)$ are respectively the number of acyclic orientations, totally cyclic orientations and spanning trees of G.) We give sufficient conditions for the inequality $T_G(x,y)T_G(y,x) \geq T_G(z,z)^2$ to hold. In particular we show that $T_G(x,0)T_G(0,x) \geq T_G(z,z)^2$ for all positive real numbers x, z with $x \geq z(z+2)$.

Friday, August 15$^{\text{th}}$, 15:30 – 16:00

János Pach, New York (USA)
Jacob Fox

Turán-type theorems for string graphs

A *string graph* is the intersection graph of a collection of continuous arcs in the plane. We consider Turán-type problems for string graphs. In particular it is shown that any string graph with m edges can be separated into two parts of roughly equal size by the removal of $O(m^{3/4}\sqrt{\log m})$ vertices. This result is then used to deduce that every string graph of n vertices with no complete bipartite subgraph $K_{k,k}$ has at most $c_k n$ edges, where c_k is a constant depending only on k. Joint work with Jacob Fox.

Thursday, August 14$^{\text{th}}$, 11:00 – 11:30

Cory Palmer, Budapest (Hungary)
Fabricio Benevides, Jonathan Hulgan, Nathan Lemons, Ago-Erik Riet and Jeffrey Paul Wheeler

Additive Properties of Two Sequences

For a given set $A \subset \mathbb{N}_0$ of non-negative integers consider the following functions

$$\begin{aligned} r(A,n) &= |\{(a_1,a_2) \in A \times A : a_1 + a_2 = n\}| \\ r_1(A,n) &= |\{(a_1,a_2) \in A \times A : a_1 + a_2 = n \text{ and } a_1 \le a_2\}| \\ r_2(A,n) &= |\{(a_1,a_2) \in A \times A : a_1 + a_2 = n \text{ and } a_1 < a_2\}| \end{aligned}$$

One well-studied problem concerning these functions is to determine necessary and sufficient conditions on A for their (eventual) monotonicity in n. In other words, for what sets A we can find an n_0 such that $r(A, n+1) \ge r(A, n)$ for all $n > n_0$? Although the three functions look similar, the conditions for their monotonicity may be quite different.

Erdős, Sárközy and T. Sós [2] proved that $r(A, n)$ is eventually monotone increasing if and only if A contains all the positive integers from a certain point on. They also obtained partial results for r_1 (independently Balasubramanian [1]) and r_2.

As a related problem, Sárközy [3] asked the following question in his excellent survey of unsolved problems in number theory.

Problem 4 in [3]. *If A, B are infinite sequences of non-negative integers, what can one say about the monotonicity (in n) of the number of solutions of the equation*

$$a + b = n,\ a \in A,\ b \in B?$$

We rephrase this question by defining a new function. For $A, B \subset \mathbb{N}_0$, let us define the *representation function* of A and B as $r(A, B, n) = |\{(a, b) \in A \times B : a + b = n\}|$.

Our main goal is to give sufficient conditions on A and B for the monotonicity (in n) of $r(A, B, n)$. We will see that this new representation function is surprisingly different from the prequel. As a tool to aid us we will develop a kind of extension of Sidon sets called co-Sidon sets. Two sets A, B of non-negative integers are called *co-Sidon* if $r(A, B) \le 1$ for all $n \in \mathbb{N}_0$. We will also discuss results in this direction that are interesting in their own right.

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Dömötör Pálvölgyi, Budapest (Hungary)
Géza Tóth

Decomposability of polygon coverings

A family of sets is a *k-fold* covering of a point set if every point is contained in at least k of the sets. A covering is *decomposable* if the sets can be partitioned into two (1-fold) coverings. We say that a geometric set is *cover-decomposable*, if there exists a k such that every k-fold covering of *any point set* in the plane by its translates is decomposable.
We show which polygons are cover-decomposable. It turns out that every convex polygon is cover-decomposable and almost every concave polygon is not cover-decomposable.

Matti Peltola, Oulu (Finland)
Hamina Martti

Structure of Least Central Subtrees of a Tree

For many purposes, one is interested in determining the "middle" of the grap h. Interesting examples are the placing of the facilities such as ambulance sta tions and firehouses. Even in the case of trees there is no such uniquely determined "middle" of a tree. The solutions are usually limited to special types of "middle part" of a tree, like central points or central paths.

In the paper [1] a new centrality concept, the subtree center of a tre e, was introduced. The concept does not restrict the structure of the "middle part" of a tree. It can be a point or a path or some other kind of subtree such that the subtree is the most central when compared with all sub trees of the tree.

For every tree T there is a joinsemilattice $L(T)$ of subtrees of T, where the meet of subtrees S_1 and S_2 equals the subtree induced by the intersection of the point sets of S_1 and S_2 whenever the intersection is nonempty and the join of subtrees S_1 and S_2 is the least subtree of T containing the subtrees S_1 and S_2. The distance in the joinsemilattice $L(T)$ is the same as the distance in the (undirected) Hasse diagram graph of $L(T)$.

A subtree S of a tree T is the central subtree of T, if S has the mi nimum eccentricity in the joinsemilattice $L(T)$. A central subtree with the minimum number of points is a least central subtree of a tree T. A least central subtree of T is the best possible connected substructure of T among all connected substructures.

We give some structural properties of a least central subtree of a tree. We describe exactly how the center and the centroid and a least central subtree of a tree are interconnected. The least central subtree of a tree is not necessarily unique. We describe how different least cental subtrees of a tree are interconnected.

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Yury Person, Berlin (Germany)
Hanno Lefmann, Vojtěch Rödl and Mathias Schacht

Extremal Problems for the Fano Plane

We present two results about the Fano plane. The first concerns the number of edge colorings a hypergraph may possess without containing monochromatic Fano planes. The second result shows that almost every Fano plane free 3-uniform hypergraph is 2-colorable.

For k-uniform hypergraphs F and H and an integer r let $c_{r,F}(H)$ denote the number of r-colorings of the hyperedges of H with no monochromatic copy of F and let $c_{r,F}(n) = \max_{H\in\mathcal{H}_n} c_{r,F}(H)$, where the maximum runs over all k-uniform hypergraphs on n vertices. Moreover, let $ex(n,F)$ be the usual *extremal* or *Turán function.*

In joint work with Lefmann, Rödl and Schacht we showed that for the hypergraph of the Fano plane F and $r = 2,3$ there exists an integer n_r, such that for every hypergraph H on $n \geq n_r$ vertices we have

$$c_{r,F}(H) \leq r^{ex(n,F)}.$$

Moreover, the only hypergraph H on n vertices with $c_{r,F}(H) = r^{ex(n,F)}$ is the extremal hypergraph for F, i.e., H is isomorphic to B_n the balanced, complete, bipartite hypergraph on n vertices. This however is no longer true for $r \geq 4$: $c_{r,F}(n) \gg r^{ex(n,F)}$.

The second question we consider concerns the asymptotic structure of "most" Fano-free hypergraphs. Together with Schacht we showed that almost every labelled, Fano-free hypergraph is 2-colorable.

Shariefuddin Pirzada, Srinagar (India)
T. A. Naikoo

Imbalances in multi digraphs

An r-digraph($r \geq 1$) is an orientation of a multigraph that is without loops and contains atmost r edges between any pair of distinct vertices. The r-imbalance of a vertex v_i in an r-digraph is defined as b_{v_i}(or simply b_i)= $d^+_{v_i} - d^-_{v_i}$, where $d^+_{v_i}$ and $d^-_{v_i}$ denote respectively the outdegree and indegree of vertex v_i. In this paper, we characterize r-imbalances in r-digraphs and obtain lower and upper bounds for r-imbalances in such digraphs. We also give the existence of an r-digraph with a given imbalance set, where an imbalance set is the set of distinct imbalances.

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András Pluhár, Szeged (Hungary)

Greedy Colorings of Uniform Hypergraphs

We give a very short proof of an Erdős [4, 5] conjecture about the size of non-2-colorable hypergraphs, originally solved by József Beck [2, 3] in 1977. Instead of recoloring a random coloring, we take the ground set in random order and use a greedy algorithm to color. The same technique works for getting bounds on k-colorability. It is also possible to combine this idea with the Lovász Local Lemma, reproving some known results for sparse hypergraphs (e.g., the n-uniform, n-regular hypergraphs are 2-colorable if $n \geq 8$, see [1]).

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Andrei Raigorodskii, Moscow (Russia)
Valentin Lyubimov

A new approach for estimating the chromatic numbers of some distance graphs

In this work, we are motivated by two classical problems of combinatorial geometry. The first problem is due to K. Borsuk who conjectured in 1933 that one can divide an arbitrary bounded non-singleton set in $\mathbb{R}^n$ into $n+1$ parts of smaller diameter (see [2], [2]). The second problem goes back to E. Nelson, P. Erdős, and H. Hadwiger. It consists in finding the chromatic number $\chi(\mathbb{R}^n)$ of the Euclidean space, i.e., the minimum number of colours that should be used to paint the space in such a way that any two points at the distance 1 apart would receive different colours (see [2], [1]).

During the last 30 years, a powerful linear algebra method has been elaborated which has been successfully used to obtain very good lower bounds for the chromatic number of $\mathbb{R}^n$ as well as to construct counterexamples to Borsuk's conjecture.

The main idea is to work with some special constructions such as *distance graphs*. Actually, a graph $G = (V, E)$ is said to be a distance graph, if $V \subset \mathbb{R}^n$, $E = \{\{\mathbf{x}, \mathbf{y}\} : \ \mathbf{x}, \mathbf{y} \in V, \ |\mathbf{x} - \mathbf{y}| = a\}$, $a > 0$. In particular, distance graphs with $V \subset \{0,1\}^n$ or $V \subset \{-1,0,1\}^n$ are of a great importance for both Borsuk's and Nelson – Erdős – Hadwiger's problems.

In a series of papers (see, e.g., [2], [2]), it was shown that the following distance graph $G = (V, E)$ could be used in order to improve substantially the known estimates for the chromatic number of the space and to reduce considerably the dimension of a counterexample to Borsuk's conjecture: $V = \{\mathbf{x} = (x_1, \ldots, x_{2k}) : \ x_i \in \{-1,0,1\}, \ |\{i : \ x_i = 0\}| = k\}, \quad E = \{\{\mathbf{x}, \mathbf{y}\} : \ \mathbf{x}, \mathbf{y} \in V, \ |\mathbf{x} - \mathbf{y}| = \sqrt{2k}\}$.
In this work, we develop a new approach for estimating the chromatic number of the graph G from above, which gives much tighter results than the usual linear algebra method. We also provide some non-trivial lower bounds. Finally, we discuss multiple applications and extensions of the approach.

The work is done under the financial support of the grant 06-01-00383 of the Russian Foundation for Basic Research, of the grant MD-5414.2008.1 of the Russian President, by the grant NSh-691.2008.1 of the Leading Scientific Schools of Russia, and by the grant of "Dynastia" foundation.

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András Recski, Budapest (Hungary)

On the algebraic representation of matroids

Representability of matroids over various fields is one of the most studied areas of matroid theory. The concepts (and even the proof of their relations) of the following sequence of statements

$$planar \subseteq graphic \subseteq regular \subseteq binary \subseteq representable$$

are standard in courses and texts on matroids.

On the other hand, many people working in classical matroid theory would not be so familiar with something like

$$regular \subseteq \sqrt[6]{1} \subseteq HPP \subseteq Rayleigh \subseteq balanced$$

although these concepts, motivated partly by some engineering applications, are interesting from the pure theoretical point of view as well.

We survey some old and new results, with special emphasis on the motivation of the new concepts.

Many of the technical details can be found in [1]. See also [2] for a more leisurely description of the engineering background.

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Monday, August 11$^{\text{th}}$, 11:00 – 11:30

Elizabeth Ribe-Baumann, Ilmenau (Germany)
Stephan Brandt

Dense Graphs with Large Odd Girth

Generalizing a result from Häggkvist and Jin [1] for graphs with odd girth at least 7, it can be shown that every graph of order n with odd girth at least $2k+1$ and minimum degree $\delta \geq 3n/4k$ is either homomorphic with C_{2k+1} or can be obtained from the Möbius ladder with $2k$ spokes via vertex duplications. The key tools used in our observations are simple characteristics of maximal odd girth $2k+1$ graphs.

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Anastasia Rozovski, Moscow (Russia)
Maria Titova, Dmitry Shabanov

On The Edge Number In Some Classes Of Uniform Hypergraphs

The talk deals with two problems in extremal hypergraph theory.

The first problem is concerned with property B_k of hypergraphs. A hypergraph is said to have property B_k if there exists a two-coloring of its vertex set such that any of its edges contains no less than k vertices of each color. The problem is to find $m_k(n)$ equal to the minimum possible number of edges of an n-uniform hypergraph that does not have property B_k. In the case $k = 1$ the problem is classical and was stated by P. Erdős and A. Hajnal in [1]. Different asymptotic bounds for $m_k(n)$ were found in [3],[4]. We deal with small values of n. It can be proved that

$$m_2(4) = 4, \quad m_2(5) = 7, \quad m_3(7) \leq 8, \quad m_4(9) \leq 8.$$

The second problem to be discussed is concerned with panchromatic s-colorings of hypergraphs. An s-coloring of hypergraph's vertex set is called panchromatic if every edge meets every of s colors. The problem is to find $p(n, s)$ equal to the minimum possible number of edges of an n-uniform hypergraph not admitting any panchromatic s-coloring. Our results improve previous (see [2]) bounds for $p(n, s)$ for some values of s. If $s \ln s = o(n)$ then

$$p(n,s) \leq \frac{1}{s}\left(\frac{s}{s-1}\right)^n \frac{en^2}{2s} \ln s(1 + o(1)).$$

There is a constant $c > 0$ such that for every $\varepsilon \in (0, 1]$ and $s \geq 2$

$$p(n,s) \geq \frac{c}{s}\left(\frac{s}{s-1}\right)^n \min\left(\frac{1}{\sqrt{s-1}}\left(\frac{n}{\ln n}\right)^{\frac{1-\varepsilon}{2}}, \left(\frac{n}{\ln n}\right)^{\varepsilon}\right).$$

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Oleg Rubanov, Moscow (Russia)
Andrei Raigorodskii

On distance graphs with large chromatic number and without large cliques

In our work, we are dealt with the classical Nelson – Hadwiger problem on finding the *chromatic number of the Euclidean space*, which is the minimum quantity $\chi(\mathbb{R}^n)$ of colours needed to paint the space in such a way that any two points at the unit distance apart receive different colours.

The notion of a *distance graph* is closely connected to the above-described problem. By a distance graph we mean such a graph $G = (V, E)$ that

$$V \subseteq \mathbb{R}^n, \quad E \subseteq \{(\mathbf{x}, \mathbf{y}) : \mathbf{x}, \mathbf{y} \in V, \ |\mathbf{x} - \mathbf{y}| = a\},$$

where a is a positive real number. For example, it is known that $\chi(\mathbb{R}^n) = \max_G \chi(G)$, where the maximum is taken over all possible finite distance graphs.

The chromatic numbers of distance graphs are well-studied. During the last decades, it has been proved that

$$4 \le \chi(\mathbb{R}^2) \le 7, \ \ 6 \le \chi(\mathbb{R}^3) \le 15, \ \ 7 \le \chi(\mathbb{R}^4) \le 49, \ \ (1.239 + o(1))^n \le \chi(\mathbb{R}^n) \le (3 + o(1))^n.$$

In 1976 P. Erdős wondered whether it was possible to find a distance graph in the plane whose chromatic number and girth would be both at least 4. In 1979 N. Wormald gave a positive answer to this question, and in 1996 P. O'Donnell and R. Hochberg succeeded in substantially improving Wormald's results. Finally, O'Donnell showed that for any fixed value k, there exists a distance graph in the plane with chromatic number 4 and girth $\ge k$.

The problem we want to discuss here is in determining the existence of a distance graph in $\mathbb{R}^n$ having simultaneously the chromatic number large enough and the clique number small enough. In particular, we can prove the two following theorems.

Theorem 1. *There exists a distance graph in $\mathbb{R}^3$ with chromatic number 5 and clique number 3, i.e., it does not contain tetrahedra.*

Theorem 2. *There exists a function $\delta(n) = o(1)$, $n \to \infty$, such that for every n, one can find a distance graph G in $\mathbb{R}^n$ having $\chi(G) \ge (\zeta_1 + \delta(n))^n$, where $\zeta_1 = 1.00297...$, and $\omega(G) \le 5$.*

Theorems 1 and 2 admit several generalizations and refinements, and we shall present them in our talk. The proofs of Theorem 1 and its extensions are constructive, whereas Theorem 2 and its relatives are obtained with the help of probabilistic and linear algebraic arguments.

The work is done under the financial support of the grant 06-01-00383 of the Russian Foundation for Basic Research, of the grant MD-5414.2008.1 of the Russian President, by the grant NSh-691.2008.1 of the Leading Scientific Schools of Russia, and by the grant of "Dynastia" foundation.

Paul Russell, Cambridge (United Kingdom)
Imre Leader and Mark Walters

Spherical sets and transitive sets

A finite subset X of $\mathbb{R}^d$ is said to be *Ramsey* if, for any number k of colours, whenever a sufficiently high-dimensional Euclidean space $\mathbb{R}^n$ is k-coloured, there exists a monochromatic isometric copy of X. It can be shown that if X is Ramsey then it can be embedded in the surface of an m-dimensional sphere for some m, and it is conjectured that the converse is also true. We present an alternative conjecture, which we show would follow from a certain Hales-Jewett-type statement.

Miklós Ruszínkó, Budapest (Hungary)
Zoltán Füredi

Regular Superimposed Codes

A **superimposed** (n, t, r) **code** is a collection $\mathcal{C}$ of subsets of an n-set with $|\mathcal{C}| = t$, such that no set is contained in the union of r others. A **superimposed** (n, t, r) **design** is a collection $\mathcal{C}'$ of subsets of an n-set with $|\mathcal{C}'| = t$, such that the unions of different at most r-tuples of subsets are different. One can easily see that an (n, t, r) design is an $(n, t, r-1)$ code, too. The degree of an element $x \in \{1, \dots, n\} = [n]$ is the number of members in $\mathcal{C}$ containing x. A superimposed (n, t, r, k) code (design) is a superimposed code (design) with maximum degree k.

Quite recently, Dyachkov and Rykov [1] introduced the concept of what they called *optimal* superimposed codes and designs. They observed [1] the following two Propositions.

Proposition 1. *For an arbitrary superimposed* $(n, t, r-1, k)$ *code (and thus for an arbitrary* (n, t, r, k) *design) with* $t > k > r \geq 2$, $n \geq \lfloor rt/k \rfloor$ *holds.*

A superimposed code (design) is called *optimal* in [1] iff in Proposition 1 equality holds. Although equality only in a very special range of parameters a superimposed code (design) may hold. Thus to avoid any confusion we will call these superimposed codes (designs) **regular** ones.

Proposition 2. *In an arbitrary regular superimposed* $(n, t, r-1, k)$ *code (and* (n, t, r, k) *design)*

- *The size of every set is* r *(*r*-uniform);*
- *The degree of every element is* k *(*k*-regular);*
- *The maximum pairwise intersection is one (*1*-intersecting).*

Dyachkov and Rykov [1] considered the case when r divides n (i.e. $n = rq$, and so $t = kq$) and obtained several sufficient conditions for the existence of regular superimposed codes and designs. Here the question is what is the minimum q which already guarantees the existence of a regular superimposed (rq, kq, r, k) code (design). Our aim is to find better bounds for $q(r, k)$, i.e. the minimum value which already guarantees the existence of a regular superimposed (rq, kq, r, k) code (design).

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Carrie Rutherford, London (UK)
Robin Whitty

Polynomial Coprimality over $GF(2)$

Suppose we are given a pair of polynomials of equal degree n, over $GF(2)$. What is the probability that they are coprime, that is, that they have no non-trivial factor in common? The surprising fact is that the probability is always precisely 1/2 [2]. This is quite a recently discovered phenomenon and it is fair to say that a good explanation for it remains to be given (although a bijection is given in [1]). Our research has revealed that, in $GF(2)$, coprimality is in some sense periodic in polynomial degree. This periodicity certainly 'explains' the probability of coprimality, although much remains to be done to formalise this argument.

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Gelasio Salazar, San Luis Potosi (Mexico)
Bernardo Ábrego, Mario Cetina, Silvia Fernández–Merchant, Jesús Leaños

The rectilinear crossing number of K_n: closing in (or are we?)

The problem of determining the rectilinear crossing number of the complete graphs K_n is an open classical problem in discrete geometry. A major breakthrough was achieved in 2003 by two teams of re searchers working independently (Abrego and Fernandez-Merchant; and Lovász, Vesztergombi, Wagner and Welzl), revealing and exploiting the close ties of this problem to other classical problems, such as the number of convex quadrilaterals in a point set, the number of $(\leq k)$–sets in a point set, the number of halving lines, and Sylvester's Four Point Problem. Since then, we have seen a sequence of improvements both from the lower bound and from the upper bound sides of the problem, and nowadays the gap between these bounds is very small. Our aim in this talk is to review the state of the art of these problems.

Friday,August 15$^{\text{th}}$ 18:00 – 18:30

Amites Sarkar, Bellingham (USA)
Paul Balister, Béla Bollobás and Mark Walters

Partitioning random geometric covers

We present some new results on partitioning both random and non-random geometric covers. For the random results, let $\mathcal{P}$ be a Poisson process of intensity one in the infinite plane $\mathbf{R^2}$, and surround each point x of $\mathcal{P}$ by the open disc of radius r centred at x. Now let S_n be a fixed disc of area $n \gg r^2$, and let $C_r(n)$ be the set of discs which intersect S_n. Write E_r^k for the event that $C_r(n)$ is a k-cover of S_n, and F_r^k for the event that $C_r(n)$ may be partitioned into k disjoint single covers of S_n. We will sketch a proof of the inequality $\mathbf{P}(E_r^k \setminus F_r^k) \leq \frac{c_k}{\log n}$, which is best possible up to a constant. Our non-random result is a classification theorem for covers of $\mathbf{R^2}$ with half-planes that cannot be partitioned into two single covers. It was motivated by a desire to understand the obstructions to k-partitionability in the original random context.

Wednesday, August 13$^{\text{th}}$, 17:30 – 18:00

Gábor Sárközy, Budapest (Hungary)
Paul Dorbec, Sylvain Gravier, András Gyárfás, Jenő Lehel, Richard Schelp and Endre Szemerédi

Cycles in hypergraphs

There are several possibilities to define cycles in hypergraphs. In this talk we survey these different cycle notions in hypergraphs and the results available for them. In particular, we introduce a new cycle definition, the t-tight Berge-cycle. We formulate the following conjecture about the existence of monochromatic Hamiltonian t-tight Berge-cycles. For any fixed $2 \leq c, t \leq r$ satisfying $c + t \leq r + 1$ and sufficiently large n, if we color the edges of the complete r-uniform hypergraph on n vertices, $K_n^{(r)}$, with c colors, then there is a monochromatic Hamiltonian t-tight Berge-cycle. We present some partial results in the direction of this conjecture.

Dmitry A. Shabanov, Moscow (Russia)

On Extremal Problems Concerning s-colorings of Hypergraphs

The talk is concerned with some generalizations of classical B property problem in extremal hypergraph theory. A hypergraph $H = (V, E)$ is said to have property $M(k, s)$ if there exists a partition $V_1, \ldots, V_s$ of the vertex set V such that for every $e \in E$ and $i = 1, \ldots, s$ the inequality $|e \cap (V \backslash V_i)| \geq k$ holds. Let $m_k(n, s)$ denote the minimum possible number of edges of an n-uniform hypergraph which doesn't have property $M(k, s)$.

The problem of finding $m_k(n, s)$ is a generalization of two other problems. The first one appears in the case $k = 1$. It is clear that a hypergraph has property $M(1, s)$ if and only if it is s-colorable. Thus, $m_1(n, s)$ is equal to the well-known value $m(n, s)$, the minimum possible number of edges in an n-uniform hypergraph, which is not s-colorable.

The second problem takes place in the case $s = 2$. Here, the property $M(k, 2)$ becomes property B_k defined in [2]. So, $m_k(n, 2)$ is equal to the value $m_k(n)$, the minimum possible number of edges of an n-uniform hypergraph which doesn't have property B_k.

Our main results are in getting lower and upper bounds for $m_k(n, s)$. If $k = O(\sqrt{\ln n})$ then for every $s \geq 2$,

$$m_k(n, s) = \Omega\left(\left(\frac{n}{\ln n}\right)^{\frac{1}{2}} \frac{s^{n-1}}{(s-1)^{k-1}\binom{n}{k-1}}\right).$$

If $k = o\left(\frac{n}{\ln n}\right)$ then for every $s \geq 2$,

$$m_k(n, s) = O\left(n^2 \ln s \frac{s^{n-1}}{(s-1)^{k-2}\binom{n}{k-1}}\right).$$

If $s = 2$, then the just-mentioned lower bound is better than the previous estimates for $m_k(n)$ (see [3]). In the case $k = 1$, we have

$$m(n, s) = \Omega\left(\left(\frac{n}{\ln n}\right)^{\frac{1}{2}} s^{n-1}\right)$$

for all $s \geq 2$. Other known bounds for $m(n, s)$ can be found in [1].

We shall also present various results concerning similar problem, in which some additional restrictions are imposed on the intersections of edges of a hypergraph.

References

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Irina Shitova, Moscow (Russia)
A. Gorskaya, V. Protassov, A. Raigorodskii

A solution to an extremum problem concerning the chromatic numbers of spaces with several forbidden distances

This talk treats of a classical question going back to E. Nelson, H. Hadwiger, P. Erdős, et al. (see [1], [2]). The question, in its more general form, is as follows: *What is the smallest quantity* $\chi(\mathbb{R}^n, \mathcal{A})$ *of colors that are essential for painting all the points in* $\mathbb{R}^n$ *so that any two points at an arbitrary distance from* $\mathcal{A} = \{a_1, \dots, a_k\} \subset \mathbb{R}_+$ *apart receive different colors?* The value $\chi(\mathbb{R}^n, \mathcal{A})$ (called *the chromatic number of the space* $\mathbb{R}^n$ *with the set* $\mathcal{A}$ *of forbidden distances*) has been studied, during the last six decades, in great detail. So first of all we shall give a brief survey of previous results.

However, the main object of this talk is the value

$$\overline{\chi}(\mathbb{R}^n; k) = \max_{\mathcal{A}:\ |\mathcal{A}|=k} \chi(\mathbb{R}^n, \mathcal{A}).$$

The best known lower estimates for this value ($k = 1, 2$) were obtained by A.M. Raigorodskii (see [2]) and I.M. Shitova (see [3]). In the joint paper [4], a method was proposed for bounding the chromatic number from below for any k, but the correspondig results were far from being optimal. In order to optimize the results, it was necessary to find a solution to a non-standard extremum problem.

In the present work, we describe such a solution and obtain lower estimates for $\overline{\chi}(\mathbb{R}^n; k)$ ($k \leq 20$), which are, in some sense, best possible. We also propose some conjectures on the growth of the quantity $\overline{\chi}(\mathbb{R}^n; k)$.

The work is done under the financial support of the grant 06-01-00383 of the Russian Foundation for Basic Research, of the grant MD-5414.2008.1 of the Russian President, by the grant NSh-691.2008.1 of the Leading Scientific Schools of Russia, and by the grant of "Dynastia" foundation.

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Gábor Simonyi, Budapest (Hungary)
János Körner and Blerina Sinaimeri

On the maximum size of graph-different permutation sets

A set of permutations of the integers $1, \ldots, n$ is called G-different with respect to some fixed graph G on the natural numbers as vertices if for any two permutations π and σ in this set there is some positive integer i for which the pair $\{\pi(i), \sigma(i)\}$ forms an edge of G.

This definition appears in [3] as a general framework in which a combinatorial puzzle of Körner and Malvenuto can be formulated in terms of asking for the maximum size of an L-different set of permutations, where L is the infinite path with edges $\{i, (i+1)\}$.

Let $T(n, G)$ denote the maximum number of permutations of $[n] = \{1, \ldots, n\}$ in a G-different set. Our main concern is the behaviour of $T(n, G)$ as a function of n for various graphs G.

If G is finite and n is large enough then $T(n, G)$ is constant and can be considered as a parameter $\kappa(G)$ of the graph G. Some initial results on the behaviour of this parameter are given in [3], cf. also [1].

For the infinite path L in the original problem of Körner and Malvenuto, the expression $\sqrt[n]{T(n, L)}$ has a limit c_L (as n goes to infinity). The value of c_L is not known. It is conjectured in [2] to be 2. Increasingly better lower bounds were obtained in [2, 3, 1].

Surprisingly, for the more complicated looking complementary graph of L we can determine the exact value of $T(n, G)$.

Theorem.

$$T(n, \overline{L}) = \frac{n!}{2^{\lfloor \frac{n}{2} \rfloor}} \quad \text{for every} \quad n \in \mathbb{N}.$$

As a corollary of this result we can also solve the problem for graphs containing as edges all pairs $\{i, j\}$ whose absolute difference is not equal to some fixed integer d.

We find graphs G with adjacency depending only on the absolute value of the difference of vertices for which the growth type of $T(n, G)$ differs from both c^n and $n!/c^n$.

The problem has some obvious relations to the Shannon capacity of graphs that we explore.

The talk is based on the paper [4].

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Monday, August 11^{th} , 12:00 – 12:30

Navin Singhi, Mumbai (India)

Finite Projective Planes and Twisted Representations

A semiadditive ring is an ordered triple $(X, +, T)$ such that $(X, +)$ is a loop with 0 as identity and (X, T) is a ternary ring satisfying, $T(x, 0, b) = T(0, x, b) = b$ for all $x, b \in X$. A semiadditive ring is said to have a (multiplicative) identity if there exists an element $1 \in X$, such that for all $a, b \in X$, $T(1, a, 0) = T(a, 1, 0) = a$ and $T(1, a, b) = a + b$.

The usual (binary) rings are examples of semiadditive rings with the ternary operation T defined by $T(x, y, z) = xy + z$. So are the well known planar ternary rings coordinatizing projective planes.

A free semiadditive ring can be defined in the usual manner. In a recent paper, it has been shown by the author that a free semiadditive ring on any set exists, is unique (up to isomorphism) and satisfies a normal form theorem. The ring of integers or a polynomial ring over the ring of integers can be thought of as a free semiadditive ring satisfying extra conditions like associativity, distributivity, commutativity and linearity. In this sense a free semiadditive ring is an analogue of the polynomial ring, when these conditions are not satisfied. A planar ternary rings is a quotient of a free semiadditive ring by a maximal ideal. The theory of semiadditive rings is being developed to create a tool to study finite planar ternary rings. A "Higman type" factorization theorem for homomorphisms of a free semiadditive ring has been proved.

Let A be a commutative ring. Let H be an additive subgroup of A. Let R be a subset of the quotient group A/H, containing $0 = 0 + H$. We choose functions $e^1 : R \to A$ and $e^2 : R \times R \times R \to A$. Let P be the ordered triple (H, e^1, e^2). Let $i = 1, 2, 3$, $\ell_i = a_i + H \in R$, $a_i \in A$. Define $T(\ell_1, \ell_2, \ell_3) \in A/H$ as follows. $T(\ell_1, \ell_2, \ell_3) = a_1a_2 + a_3 + e^1(\ell_1)a_1 + e^1(\ell_2)a_2 + e^2(\ell_1, \ell_2, \ell_3) + H$

Suppose T is a well defined ternary operations on R, then we will say that operation T is obtained by twisting operations in the ring A with twisting triple P. Suppose T is an operation with a multiplicative identity 1, i.e., $T(1, x, 0) = T(x, 1, 0) = x$ for all $x \in R$. We then define a binary operation $\oplus$ on R, by $x \oplus y = T(1, x, y)$. If $(R, \oplus, T)$ is a semiadditive ring, we say that the semiadditive ring R is obtained by twisting operations in A with twisting triple P.

Now suppose R_1 is a semiadditive ring and f is an isomorphism of R_1 onto the semiadditive ring $(R, \oplus, T)$. We will say that R_1 has a twisted representation f in the ring A, with the twisting triple P. If $R = A/H$, we will say that the twisted representation is complete.

It can be easily seen that every semiadditive ring A (with a multiplicative identity), has a representation in any commutative ring of size bigger than A. The well known Albert's twisted field has a complete representation in a field with $H = (0)$. If a planar ternary ring has a complete representation in a field, clearly it will be of size a power of a prime number. Some twisted representations of semiadditive rings in a polynomial ring over the ring of integers, created using the above mentioned results will be described in the talk.

Jozef Skokan, London (UK)
Lubos Thoma

The Ramsey numbers for hypergraph cycles.

Denote by $\mathcal{C}_n$ the 3-uniform hypergraph *loose cycle*, that is the hypergraph with vertices $v_1, \ldots, v_n$ and edges $v_1v_2v_3$, $v_3v_4v_5$, $v_5v_6v_7, \ldots, v_{n-1}v_nv_1$. Haxell et al [1] proved that every red-blue colouring of the edges of the complete 3-uniform hypergraph with N vertices contains a monochromatic copy of $\mathcal{C}_n$, where N is asymptotically equal to $5n/4$. We determine this number N for large values of n.

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László A. Székely, Columbia SC (USA)
Mike Steel

Phylogenetic and classical combinatorics

A phylogenetic tree is a binary tree in which the leaves are labelled with different labels. A binary subtree of a phylogenetic tree is obtained by selecting a subset of the leaf vertices, taking their spanning subtree, and in the spanning subtree contracting recursively edges, in which at least one endvertex has degree 2. A well-known fact is, that given two different phylogenetic trees with n leaves each, using the same label set, there should be a 4-leaf binary tree, which is a binary subtree of one of the phylogenetic trees, but not of the other. The interesting question is whether additional requirements can be made on the number of vertices of degree 2 on the paths in the spanning subtrees (in the definition of the binary subtree, before contractions).

The extremal version of the Maximum Agreement Subtree Problem asks how large common binary subtree must be always there for two phylogenetic trees with n leaves each, using the same label set. These problems show analogy with Ramsey theory and come up naturally in phylogeny reconstruction.

Friday, August 15^{th} , 12:00 – 12:30

Balázs Szörényi, Szeged (Hungary)

Disjoint DNF Tautologies with Conflict Bound Two

A decision tree naturally encodes a DNF tautology—each term of which corr esponds to a unique leaf of the tree—, which has the following special pr operties: (a) the terms are pairwise conflicting, and (b) the terms possess a hierarchical structure. Such a DNF is called a *DT-DNF* (*decision tree generated DNF*), meanwhile a DNF possessing property (a) but not necessarily property (b) is called a *D-DNF* (*disjoint DNF*). The relationship between DNF tautologies and decision trees was investigate d by Lovász *et al.* in [4]. More precisely they were interested in the following (search) problem: give n a DNF tautology F, the task is to construct a decision tree T such th at each term of the DNF generated by T has a subterm appearing in F. They have shown that for some "very small" DNF tautologies this problem c an be solved only with "extremely large" decision trees.

On the other hand, as it has been proved by Kullmann [2] (and, independently by Sloan *et al.* [3]), when restricting the DNFs to the subclass possessing property (a) (i. e., the class of D-DNFs), *and* further bounding the number of conflic ts between the terms to one (i.e., for each pair of terms there is *ex actly* one variable appearing negated in one of them and unnegated in the o ther), it turns out that the resulting class consists of DNFs that can all be generated by decision trees.

This problem arose in connection with characterizing strongly minimal tauto logies with the additional property that the number of terms is one more than the number of variables [1, 2], and also in connection with maximal DNFs [3]. Here we prove the following strengthening of the above result of Kullman:

Theorem.

If F is a D-DNF tautology with terms conflicting in one or two variables pairwise, then F is a DT-DNF.

For larger conflict numbers such a statement does not hold. We formulate a related general combinatorial problem on partitions of the hypercube into subcubes. [1]

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Wednesday, August 13$^{\text{th}}$, 18:00 – 18:30

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Dirk Oliver Theis, Brussels (Belgium)
Gwenaël Joret, Marcin Kamiński

The Cops & Robber game on graphs with a forbidden (induced) subgraph

The Cops and Robber game (Nowakowski & Winkler [2], Quilliot [3]) is a two-player game played on undirected finite graphs. k cops and one robber are positioned on vertices and take turns in sliding along edges. The cops win if, after a move, a cop and the robber are on the same vertex. For a fixed finite graph, the minimum over all numbers k such that the cop player has a winning strategy is called the cop number of the graph.

Andreae [1] showed that any class of graphs defined by forbidding a fixed graph as a minor has bounded cop number.

In this talk, we discuss the question whether classes of graphs defined by forbidding one or more graphs as either subgraphs or induced subgraphs have bounded cop number. In the case of a single forbidden graph, for both relations, we completely characterize the graphs which force the cop number to be bounded.

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Norihide Tokushige, Okinawa (Japan)
Hiroshi Maehara

A regular tetrahedron passes through a hole smaller than its face

No triangular frame can hold a convex body, and a convex body can pass through a triangular hole Δ if and only if the convex body can be congruently embedded in a right triangular prism with base Δ. Applying these result, one can show the following: a regular tetrahedron of unit edge can pass through an equilateral triangular hole if and only if the edge length of the hole is at least $(1+\sqrt{2})/\sqrt{6} \approx 0.9856$.

I will also mention some related results in higher dimensions, e.g., an n-dimensional unit hypercube can contain a regular n-simplex of edge length $n^{1/2-\delta}$ for any $\delta > 0$ and $n > n_0(\delta)$.

Tuesday, August 12^{th} , 18:00 – 18:30

Andrew Treglown, Birmingham (UK)

Hamiltonian degree sequences in digraphs

Since it is unlikely that there is a characterization of all those graphs which contain a Hamilton cycle it is natural to ask for sufficient conditions which ensure Hamiltonicity. One of the most general of these is Chvátal's theorem that characterizes all those degree sequences which ensure the existence of a Hamilton cycle in a graph: Suppose that the degrees of a graph G are $d_1 \leq \cdots \leq d_n$. If $n \geq 3$ and $d_i \geq i+1$ or $d_{n-i} \geq n-i$ for all $i < n/2$ then G is Hamiltonian. This condition on the degree sequence is best possible in the sense that for any degree sequence violating this condition there is a corresponding graph with no Hamilton cycle.

Nash-Williams [2] raised the question of a digraph analogue of Chvátal's theorem quite soon after the latter was proved. I will discuss the following approximate version [1] of this conjecture: Given any $\eta > 0$ every digraph G of sufficiently large order n is Hamiltonian if its out- and indegree sequences $d_1^+ \leq \cdots \leq d_n^+$ and $d_1^- \leq \cdots \leq d_n^-$ satisfy (i) $d_i^+ \geq i + \eta n$ or $d_{n-i-\eta n}^- \geq n-i$ and (ii) $d_i^- \geq i + \eta n$ or $d_{n-i-\eta n}^+ \geq n-i$ for all $i < n/2$. In fact, such digraphs G are pancyclic. This is joint work with Daniela Kühn and Deryk Osthus.

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Monday, August 11$^{\text{th}}$, 18:00 – 18:30

György Turán, Chicago (USA), Szeged (Hungary)
Marina Langlois, Dhruv Mubayi and Robert H. Sloan

Combinatorial Problems for Horn Formulas

We consider combinatorial problems for propositional Horn formulas, i.e., expressions like

$$(a, b \to c) \wedge (a, c \to d) \wedge (d, e \to f).$$

Horn formulas are an expressive fragment of propositional logic, and several basic computational problems, such as satisfiability, are efficiently solvable for them. Therefore, Horn formulas are a basic framework for many applications in artificial intelligence and computer science. *Resolution* applied to the first two clauses in the example gives the resolvent clause $(a, b \to d)$, and applied to the last two clauses gives the resolvent clause $(a, c, e \to f)$. Resolution is a sound and complete method to derive implications of clauses.

We consider definite Horn clauses of size 3 (like in the example above) over n variables, and look at the following questions. What is the minimal number of Horn clauses implying all other clauses in the family? What is the maximal number of clauses from the family without any resolvents (resp., any resolvents of size 3, or of size 4)? Is there a phase transition for the probability that a random subfamily of a given size implies all the other clauses? Sharp bounds are given answering these questions. Some of the proofs use extremal results for graphs and hypergraphs. Several open problems are formulated.

This work is motivated by our previous work on knowledge compilation [1] and on the *KnowBLe (knowledge base learning)* problem (learning a Horn knowledge base using a rational hypothesis updating algorithm) [2].

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Venkaiah V. Ch. Venkaiah, Hyderabad (India)
Kishore Kothapalli, Bharat Joshi, Mahesh Raja, and K. Ramanjaneyulu

Simple Acyclic Edge Coloring Algorithms for a Class of Complete Graphs

An edge colouring of a graph G is proper if no two incident edges have the same colour. It is acyclic if it is proper and does not induce any bichromatic cycle. The acyclic edge chromatic number of a graph G, denoted $a'(G)$, is the minimum number of colors in an acyclic edge colouring of G. Alon et al. [1] show that it is possible to acyclically edge colour G using at most 64Δ colours. Alon et al. [2] claim that the constant can be improved further and also conjectured that $a'(G) \leq \Delta + 2$. They prove their conjecture, partially, for graphs with girth at least $c\Delta \log \Delta$ for a constant c. Muthu et. al. [3] improve this bound to at most 6Δ colours for graphs with girth at least 9. However, all the above results are based on probabilistic arguements using the Lovasz Local Lemma. There has been very little algorithmic study on acyclic edge colouring except for the following. Skulrattankulchai [4] presented a linear time algorithm for acyclically edge coloring sub-cubic graphs using at most 5 colors. Alon et al. [1] gave an algorithm that can acyclically color any complete graph on a prime number, p, of vertices using p colors. They also present an algorithm that can acyclically edge color a complete bipartite graph $K_{p-1,p-1}$, where p is prime, using p colors. Using known results about the distribution of primes, it may be inferred [1] that $a'(K_n) \leq n + O(n^{2/3})$ and $a'(K_{n,n}) = n + O(n^{2/3})$. In this abstract, we present an algorithm to acyclically color a complete graph K_n where $n = p(q-1)$, p, q prime, using pq colors. This result is based on the work of Alon et al. [1]. The main idea is to treat $K_{p(q-1)}$ as a complete (multi)graph on p vertices where each vertex corresponds to a complete graph on $(q-1)$ vertices. Now, this complete graph on p vertices can be colored using p colors [1]. Similarly, K_{q-1} can be coloured using at most q colors. Treating each multiedge in the K_p as $K_{q-1,q-1}$, this can be colored using at most q colors [1]. This now can be used to acyclically edge color K_n using pq colors which improves the result of [1]. Next, we present an algorithm to acycically edge color K_n using at most $2n-3$ colors. The idea is to use color $i+j-2$ as the color of the edge (i,j). This may be modified to color K_p, p prime so that the color of the edge (i,j) is $i+j-2 \bmod p$. The resulting coloring thus uses p colors for acyclically edge coloring K_p, p prime. Also, we experimented the validity of Alon et al. [2] conjecutre using the above mentioned algorithm on complete graphs. Our experiment uses $i+j-2 \bmod K$ as the color of the edge (i,j) with K starting from $\Delta + 1$. The value of K is incremented by 1 if the current number of colors do not suffice to arrive at an acyclic edge coloring. The results were encouraging.

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Andrew Vince, Gainesville (USA)
Hua Wang

Two Conjectures Concerning Trees

In 1928 Sperner proved that the boolean lattice has the property that a maximum antichain consists of elements from a single rank. A partially ordered set that satifies this property is now called a *Sperner poset*. The set $C(T)$ of subtrees of a tree T is a lattice with respect to the includion ordering, the rank of a subtree being its order. Jacobson, Kézdy and Seif asked in 1992 whether there exists an infinite family of trees T such that the subtree poset $C(T)$ of each tree in the family fails to have the Sperner property. Such a family is provided in this talk.

An explicit formula for the number of elements of $C(T)$ of an arbitrary rank (Whitney number) is not known. However, for a tree T all of whose internal vertices have degree at least three, Jamison conjectured in 1983 that the average order of a subtree is at least half the order of T. We show that this is so and, in addition, that the average order of a subtree of T is at most three quarters the order of T.

Tuesday, August 12^{th} , 15:30 – 16:00

Stephan Wagner, Stellenbosch (South Africa)
Éva Czabarka and László Székely

The inverse problem for certain tree parameters

Let p be a graph parameter that assigns a positive integer v= alue to every=20 graph. The inverse problem for p asks for a graph within a prescribed clas= s (this talk is only concerned with trees), given the value of p. This is of interest in combinatorial chemistry, where graph parameters are used as molecular descriptors, see e.g. [2]. In this context, it is of interest to know whether such a graph can be=20 found for all or at least almost all integer values of p. We will=20 discuss a general setting for this type of problem over the set of=20 all trees and describe some simple examples. The following two theorems can be proved by means of an explicit construction together with elementary number-theoretic considerations:

1. Every positive integer, with only 49 exceptions, is the Wiener index (= i.e., the sum of all distances between pairs of vertices) of some tree. [3, 4]

2. Every positive integer, with only 34 exceptions, is the number of subt= rees of some tree. [1]

ed by the examples below.

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Peter Wagner, Rostock (GERMANY)
Andreas Brandstädt

Leaf powers - an overview

Motivated by a biological background, Nishimura, Ragde and Thilikos [6] introduced the notion of *k-leaf power* and *k-leaf root.* Let $G = (V, E)$ be a finite simple graph; that is, an undirected graph with finite vertex set, no self-loops and no more than one edge between any two distinct vertices. For $k \geq 2$, a tree T is a *k-leaf root* of G if V can be identified as the leaf set of T and, for any two distinct vertices $x, y \in V$, x and y are adjacent in G if and only if their distance in T is at most k; that is, $xy \in E \iff d_T(x, y) \leq k$. G is a *k-leaf power* if it has a k-leaf root.

Since then, a lot of work has been done on k-leaf powers and roots as well as on their variants phylogenetic roots and Steiner roots. For $k = 3$ [1] and $k = 4$ [2], structural characterisations of and linear time recognition algorithms for the class of k-leaf powers are known, and, recently, a linear time recognition of the class of 5-leaf powers was given [5]. For larger k, the recognition problem is open.

In this talk, we shall give a current overview of the topic, including a discussion of the modification of (k, ℓ)-leaf powers [3] and new results about the comparability of the various k-leaf power classes [4].

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Janez Žerovnik, Ljubljana (Slovenia)
joint work with Iztok Banič and Rija Erveš

Wide and fault diameters of Cartesian graph bundles

Fault tolerance and transmission delay of networks are important concepts in network design. The notions are strongly related to connectivity and diameter of a graph, and have been studied by many authors. Wide diameter of a graph combines studying connectivity with the diameter of a graph. Diameter with width k of a graph G is defined as the minimum integer d for which there exist at least k internally disjoint paths of length at most d between any two distinct vertices in G. In the context of computer networks, wide diameters of Cartesian graph products have been recently studied [4, 5]. Cartesian graph bundles [6] is a class of graphs that is a generalization of the Cartesian graph products. We show that if G is a k_G-connected graph and $D_c(G)$ denotes the c-diameter of G, then $D_{a+b}(G) \leq D_a(F) + D_b(B)$, where G is a graph bundle with fiber $F \neq K_2$ over base $B \neq K_2$, $0 < a \leq k_F$, and $0 < b \leq k_B$ [3]. Not surprisingly, there are analogous inequalities known for some related invariants including vertex- and edge-fault diameters [2, 1] and hence it is interesting to study the relationships among these obviously related notions.

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