ALFRÉD RÉNYI INSTITUE OF MATHEMATICS

ASYMPTOTIC GROUP THEORY

PÉTER PÁL PÁLFY IS 60

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ABSTRACTS

László Babai: Symmetry versus regularity

Symmetry is defined in terms of automorphisms; regularity in terms of numerical parameters. Distance-transitive vs. distance-regular graphs illustrate this distinction. Symmetry always implies regularity; the converse is false in general. Regularity, somewhat paradoxically, is known to often limit symmetry. We mention two of the speaker's old asymptotic conjectures in this direction. We say that a function $f(n)$ grows *exponentially* if for some positive ϵ and all sufficiently large n we have $f(n) > \exp(n^{\epsilon})$; and $g(n)$ grows *subexponentially* if for all $\epsilon > 0$ and all sufficiently large n we have $g(n) < \exp(n^{\epsilon})$.

Conjecture 1. With known (easy) exceptions, the number of automorphisms of a strongly regular graph with n vertices is subexponential in n.

Conjecture 2. If a primitive coherent configuration with n vertices has exponentially many automorphisms, then, for sufficiently large n, its automorphism group is primitive (and therefore known, due to a 1981 classification of large primitive permutation groups by Peter Cameron that relies on the Classification of Finite Simple Groups).

We describe progress on these and related questions. This body of work is motivated in part by a classification-free approach to Cameron's result, and in part by the complexity of the graph isomorphism problem. It has some connection to a 1982 result by Cameron, Pálfy, and the speaker.

The recent results to be surveyed are due to various subsets of Xi Chen, Xiaorui Sun, Shang-Hua Teng, John Wilmes, and the speaker; significant part of the credit goes to a 1996 paper by Daniel Spielman.

Alexandre Borovik: The Black Box Philosophy

(joint work with Sukru Yalcinkaya)

I will outline some general methodological principles behind the results on black box groups presented in Sukru Yalcinkaya's talk.

Peter Cameron: Numerical and graphical invariants of finite groups

The generating graph of a finite group G has received a lot of attention recently. Motivated by the fact that the generating graph has an unexpectedly large automorphism group, we define a reduced generating graph which is more closely related to G. This is specific to 2-generated groups, but we define a descending chain (under refinement) of equivalence relations on G, such that the second relation is the one used in the reduction just described. The number of steps until the chain stabilises is an interesting invariant, related to the rank (the minimal size of a generating set), the maximum size of a minimal generating set, and the maximum of the ranks of the maximal subgroups of G. These parameters are in turn related to the subgroup lattice and to the maximal size of an irredundant or minimal base in a permutation action of G.

The most recent work reported here is joint with Colva Roney-Dougal.

Rachel Camina: Influences of conjugacy class sizes on finite groups

Given the conjuacy class sizes of a finite group which structural properties of the group can be deduced? We will look at different results in this area.

INNA CAPDEBOSCQ: FINITE SIMPLE GROUPS OF (EVEN TYPE AND) MEDIUM SIZE

In this talk we will discuss classification of finite simple groups of even type of medium size. This is a part of the ongoing project of Gorenstein, Lyons and Solomon (the Generation-2 proof of the Classification of Finite Simple Groups).

ELOISA DETOMI: ON INVARIABLE GENERATION OF GROUPS

A group G is *invariably generated* by a subset S of G if $G = \langle s^{g(s)} \mid s \in S \rangle$ for each choice of $g(s) \in G$, $s \in S$. We show that the free prosoluble group of rank $d \geq 2$ cannot be invariably generated by a finite set of elements, while the free solvable profinite group of rank d and derived length l is invariably generated by precisely $l(d-1)+1$ elements. We will also discuss the structure of finite groups invariably generated by a set whose elements have coprime orders or coprime prime-power orders.

ZOLTÁN HALASI: BASE SIZES FOR GROUP ACTIONS

The base sizes for permutation groups has a great importance in the theory of permutation groups, and it is widely examined in the past. In this talk we present some results concerning the base size both for permutation groups in general and for linear groups in particular. Some recent results in representation theory using our joint work with K. Podoski and with A. Maróti about base sizes of coprime and p-solvable linear groups will also be mentioned in the talk.

Pál Hegedűs: Positive aspects of Hilbert's 14th problem

Abstract: Hilbert's 14th problem was the following: If K is a field, $K[x_1, \ldots, x_n]$ is a polynomial ring over K. If $K \subseteq L \subseteq K(x_1, \ldots, x_n)$ is a subfield then $L \cap K[x_1, \ldots, x_n]$ is finitely generated over K.

In full generality it turned out to be false, the first counterexample was provided by Nagata in 1958. The problem was originally motivated by the case when L is the field of invariant rational functions of a linear group action. Another case which covers much of the original problem is when L is the field of rational constants of a derivation. Nagata's counterexample (and most subsequent ones) can be fit into both approaches. In the talk I will describe affirmative theorems in both ways of understanding the problem. One is a joint work with Laci Pyber, the other is with Janusz Zieliński.

Marty Isaacs: On the number of elements of a group that are not p-th powers

Fix a prime p and suppose that a finite group G with order divisible be p has exactly n elements that are not p-th powers in G. We show that $|G| \leq n(n+1)$, and that if equality occurs, the structure of G is highly constrained. Some related questions will also be discussed.

Mikhail Klin: Yes, a non-Schurian coherent configuration on 14 points exists

(joint work with Matan Ziv-Av)

This project belongs to Algebraic Graph Theory (briefly AGT). We will start with an informal discussion of some of the main paradigms of AGT, which are related to the concept of a coherent configuration (briefly CC) and its particular case of an association scheme. Among the founders of this part of AGT are Boris Weisfeiler and Andrei Leman, as well as Donald Higman. A CC is called Schurian if it appears in a standard way from a suitable permutation group. The smallest known non-Schurian CCs are some association schemes on 15, 16 and 18 points. For a few decades existence of a smaller non-Schurian CC was an open question.

Essential use of a computer allowed us to enumerate all CCs on up to 13 points and to prove that they are Schurian. We also discovered a rank 11 non-Schurian CC on 14 points with two fibers of size 6 and 8. Its automorphism group has rank 12, order 24 and is isomorphic to the binary tetrahedral group. A computer free interpretation of this new CC will be discussed.

ALEX LUBOTZKY: HIGH DIMENSIONAL EXPANDERS AND RAMANUJAN COMPLEXES

Ramanujan graphs are optimal expanders (from spectral point of view). Explicit constructions of such graphs were given in the 80's as quotients of the Bruhat-Tits tree associated with $\mathrm{GL}(2)$ over a local field F, by suitable congruence subgroups. The spectral bounds were proved using works of Hecke, Deligne and Drinfeld on the "Ramanujan conjecture" in the theory of automorphic forms. The work of Lafforgue, extending Drinfeld from $\mathrm{GL}(2)$ to $\mathrm{GL}(n)$, opened the door for the construction of Ramanujan complexes as quotients of the Bruhat-Tits buildings. This gives finite simplical complxes which on one hand are ”random like” and at the same time have strong symmetries. Recently various applications have been found in combinatorics, coding theory and in relation to Gromov's overlapping properties. We will describe these developments and some recent applications. In particular, we will present a joint work with Tali Kaufman and David Kazhdan in which these complexes are used to (partially) answer a question of Gromov.

Andrea Lucchini: Bias of group generators in the solvable case

Babai and Pak demonstrated a weakness in the product replacement algorithm, a widely used heuristic algorithm intended to rapidly generate nearly uniformly distributed random elements in a finite group G. It was an open question whether the same weakness can be exhibited if one considers only finite solvable groups. We give an affirmative solution to this problem. We consider the distribution of the first component in a k-tuple chosen uniformly in the set of all the k-tuples generating G and construct an infinite sequence of finite solvable groups G for which this distribution turns out to be very far from uniform.

Marta Morigi: Coverings of word values

A word w is an element of the free group on n generators. If G is a group, we may see w as a function $w : G^n \to G$, and we denote by G_w the set of values taken by w. The verbal subgroup $w(G)$ is the subgroup generated by G_w. We will address the following question: if G_w is contained in the union of a finite number of subgroups of G all satisfying some property, what information can we deduce about $w(G)$? Particular emphasis will be put on the case when G is a profinite group and w is a multilinear commutator word. We will also suggest a new definition of conciseness for words in the class of profinite groups

Michael Muzychuk: On code equivalence problem for group codes

Let $\mathbb{F}_q[H]$ be a group algebra of a finite group H over Galois field $\mathbb{F}_q$. A *group code* is a right ideal I of $\mathbb{F}_q[H]$. It is called semisimple if $\gcd(q, |H|) = 1$. Two group codes $I, J \subseteq \mathbb{F}_q$ are called (*permutation*) *equivalent*, notation $I \sim J$, iff there exists a permutation $g \in \mathrm{Sym}(H)$ such that $I^g = J$. If there exists $g \in \mathrm{Aut}(H)$ mapping I to J, then we say that the codes are *Cayley* equivalent. A group H is called a $\mathcal{C}$-CI-group iff Cayley equivalent codes are the only permutation equivalent codes. In my talk I'll present several results about $\mathcal{C}$-CI-groups. One of them states that a cyclic group is a $\mathcal{C}$-CI-group iff its order is not divisible by 8 and by an odd square.

Gabriel Navarro: On real finite groups
(joint work with P. Tiep)

We study a conjecture of Rod Gow on reality.

Nikolay Nikolov: On the growth of torsion in homology of finite index subgroups

There is a lot of interest regarding the growth of invariants of chains of finite index subgroups, e.g. the growth of Betti numbers, rank, deficiency and so on. In this talk I will consider the growth of another invariant: the size of the torsion subgroup in homology. I will focus on two main classes of groups where there has been recent progress: amenable groups (joint with Kar and Kropholler) and right angled groups (joint work with Abert and Gelander). The main tools are from combinatorial group theory and the notion of combinatorial cost.

Ilia Ponomarenko: On permutation groups determined by the intersection numbers of associated coherent configuration

Let G be a permutation group on a set Ω of size n. Then there exists an integer $m \leq n$ such that G is determined up to permutation group isomorphism by the intersection numbers of the coherent configuration associated with the action of G on Ω^m. The minimal m with this property is often small when the rank of G is sufficiently large in comparison with the maximal subdegree of G. This phenomena is demonstrated for some classes of groups including simple groups acting on the cosets of the Cartan subgroup (a recent joint result of the author and A.Vasil'ev).

REINHARD PÖSCHEL: AUTOMORPHISM GROUPS OF SCHUR RINGS OVER CYCLIC GROUPS OF PRIME-POWER ORDER

(joint work with Mikhail Klin)

Schur rings are a well-known tool for the investigation of permutation groups and associated algebraic and combinatorial structures. The automorphism groups of Schur rings are of special interest because they describe "symmetries" of combinatorial Cayley objects, e.g. circulant graphs (i.e., Cayley graphs over a cyclic group Z_n) and provide efficient isomorphism criteria for such Cayley objects (here we meet connections to results of P^3). In this talk we give some insight into the long history of determining the automorphism group of Schur rings, in particular for Schur rings over the cyclic groups Z_{p^m} of prime-power order. We describe so-called subwreath products of permutation groups and show how this approach can lead to an explicit and constructive presentation of the automorphism group of an arbitrary Schur ring over Z_{p^m} and to isomorphism criteria for circulant graphs.

JAN-CHRISTOPH SCHLAGE-PUCHTA: THE SUBGROUP GROWTH OF THE NOTTINGHAM GROUP

(joint work with Yiftach Barnea and Benjamin Klopsch)

For a finitely generated pro-p group G, let $s_{p^n}(G)$ be the number of subgroups of index p^n, and define $\beta(G)$ as $\limsup \frac{\log s_{p^n}(G)}{n^2}$. Shalev showed that either $\beta(G) \geq \frac{1}{8}$, or G is p-adic analytic, in particular $\beta(G) = 0$. Mann asked, what $\beta = \inf\{\beta(G) : \beta(G) > 0\}$ is. Barnea and Guralnick showed that $\beta(\mathrm{Sl}_2(\mathbb{F}_p[[t]])) \leq \frac{1}{2}$, thus $\frac{1}{8} \leq \beta \leq \frac{1}{2}$.

We show that for sufficiently large p the Nottingham group J_p satisfies $\beta(J_p) = \frac{1}{8}$. The proof involves a new type of question in the area of additive combinatorics. It appears that similar ideas can be applied to other $\mathbb{F}_p[[t]]$-analytic groups as well, however, the technical details become quite complicated.

YOAV SEGEV: A NON-SPLIT SHARPLY 2-TRANSITIVE GROUP

(joint work with E. Rips and K. Tent)

In a pair of landmark papers from 1936 Zassenhaus gave a complete classification of the FINITE sharply 2-transitive (hence-forth s2t) groups. The first of these papers shows that every such group can be identified with the group of all affine transformations of the form $\{x \mapsto ax + b \mid a \in F^*, \text{ and } b \in F\}$, where F is a FINITE near-field; and this fact is equivalent to the assertion that every FINITE s2t group SPLITS, i.e., it has a non-trivial abelian normal subgroup. The second classified all FINITE near-fields.

The answer to the question of whether any INFINITE s2t group splits defied the attempts of many mathematicians.

We give (the first) example of an infinite non-split s2t group. Indeed we show that ANY GROUP can be embedded into a non-split s2t group.

ANER SHALEV: FIXED POINTS AND WHAT THEY ARE GOOD FOR

I will discuss fixed points of elements of permutation groups, presenting background and latest developments. I will then focus on applications to various topics, such as base size and invariable generation.

Gábor Somlai: CI property for Cayley maps

(joint work with Mikhail Muzychuk)

Cayley maps are 2-cell embeddings of Cayley graphs into orientable surfaces with the extra property that the vertices have the same cyclic orientation at each vertex. A Cayley map can also be considered as a ternary relational structure. The Cayley isomorphism (CI) property is well studied for graphs and a theorem of Pálfy says that in order to describe CI-groups for any n-ary relational structures we may assume $n \leq 4$.

The Cayley isomorphism problem for ternary relational structures was investigated by Dobson and Spiga and they give an almost complete list of CI-groups for ternary relational structures and proved that the class of CI-groups with respect to ternary relational structures is narrow. We present a similar partial solution for CI-groups with respect to maps.

Balázs Szegedy: Nilpotent groups in combinatorics

Szemerédi's theorem is one of the most famous results in additive combinatorics. It inspired research in various fields of mathematics. In the past few decades a surprising connection emerged between Szemerédi's theorem (together with many similar problems in additive combinatorics) and actions of nilpotent Lie groups. This connection was crucial in the development of higher order Fourier analysis as initiated by W. T. Gowers. It also initiated new research in the classical field of nilpotent groups and lead to a new point of view. In this talk we give an overview of this subject.

EVGENY VDOVIN: INTERSECTION OF SOLVABLE SUBGROUPS IN FINITE GROUPS

In 2010 the author of the talk added to "Kourovka notebook" the following problem:

Problem 17.41. *Let S be a solvable subgroup of a finite group G that has no nontrivial solvable normal subgroups.*

(a) (L. Babai, A. J. Goodman, L. Pyber) Do there always exist 7 conjugates of S whose intersection is trivial?

(b) Do there always exist 5 conjugates of S whose intersection is trivial?

In our talk we discuss the recent progress in solution of the problem.

SUKRU YALCINKAYA: TWENTY YEARS OF ATTACKS ON UNIPOTENT ELEMENTS

(joint work with Alexandre Borovik)

The construction of a unipotent element in groups of Lie type is the fundamental part of the constructive recognition algorithms in black box group theory. In this talk, I will reduce the problem of constructing a unipotent element in a black box group of Lie type of odd characteristic to the groups $\mathrm{PGL}(2, q)$. Then, by using the geometry of involutions in $\mathrm{PGL}(2, q)$, I will talk about the construction of the underlying black box projective plane and black box field. This procedure readily gives an efficient (in time polynomial in the input length) algorithm which constructs a unipotent element in black box groups of Lie type of odd characteristic. I will also explain how our algorithm works for the groups $\mathrm{PGL}(2, q)$ in characteristic 2.

EFIM ZELMANOV: THE SPECHT PROBLEM AND PRO-p GROUPS

We will discuss a version of The Specht Problem and it's implications for identities and verbal width of pro-p and prounipotent groups.

JIPING ZHANG: SPECIAL BLOCKS OF FINITE GROUPS

We will determine the structure of the generalized Fitting subgroup $F^*(G)$ of the finite groups G all of whose defect groups (of blocks) are conjugate under the automorphism group $Aut(G)$ to either a Sylow p-subgroup or a fixed p-subgroup of G. Then we prove that if a finite group H acts transitively on the set of its proper Sylow p-intersections then either $H/O_p(H)$ has a T.I. Sylow p-subgroup or $p = 2$ and the normal closure of a Sylow 2-subgroup of H is 2-nilpotent with completely descripted structure. This solves a long-open problem. We also obtain some generalizations of the results on half-transitivity.