Preface

Personal reminiscences

I met Paul Turán first in September 1930 at the University of Budapest though we knew of each others existence since we both worked for the mathematical journal for high school students and our first joint paper appeared there, i.e. a solution of a problem which we obtained independently. At our first meeting I asked him if the sum of the reciprocals of the primes diverges or converges. He informed me that it diverges and he told me about the prime number theorem. (Seven or eight years earlier I learned from my father that the number of primes is infinite.)

We met and discussed mathematics nearly every day until I went to Manchester in October 1934, after that date until early September 1938 I spent half my time in England half in Hungary but when I was in England we corresponded a great deal. Our first joint paper dealt with elementary number theory, in which we deal with a problem of G. Grünwald and D. Lázár: Let $f(n)$ be the largest integer so that if $1 \leqq a_1 < \ldots < a_n$ are any set of n integers then $\prod_{1 \leqq i \leqq j \leqq n} (a_i + a_j)$ has at least $f(n)$ distinct prime factors.

We proved

$$c_1 \log n < f(n) < c_2 \frac{n}{\log n} \tag{1}$$

and we conjectured that $f(n)/\log n \to \infty$, which is still open.

Very soon we started our collaboration on interpolation which produced many more "serious" results.

From England I returned to Hungary for Christmas, Easter and summer vacations. In the spring of 1938 Hitler succeeded in disturbing my plans, but I could return to Hungary precariously in the summer of 1938. On September 3, 1938, I did not like the news and in the evening I was on my way to England and three and one-half weeks later to the USA. We corresponded until 1941, then there was an enforced gap of four years. As soon as possible we started our first postwar joint paper on the difference of consecutive primes which we wrote in correspondence. In this paper we stated a few problems which I find very interesting: Put $d_k = p_{k+1} - p_k$. Is it true that

$$d_k > d_{k+1} > d_{k+2} \quad \text{and} \quad d_k < d_{k+1} < d_{k+2}$$

both have infinitely many solutions? We also observed that we cannot prove that there is no k_0 for which

$$(-1)^j(d_{k_0+j}-d_{k_0+j-1}), \qquad j=1,2,\ldots$$

always has the same sign. I am of course sure that such a k_0 does not exist.

We met in the USA and Hungary in 1948–49 and wrote two papers on equidistribution, both of which I believe will outlive the authors. From that date we always corresponded but we could not meet until 1955. Our work on statistical group theory started around 1960 and in some sense is still continuing. We also worked with V. T. Sós (Mrs. Turán) and A. Meir (Meyer), a student of Turán, on applications of combinatorial analysis to geometry and various branches of analysis.

I knew that he was not long for this world but hoped to see him at Christmas. Our last mathematical discussion took place two weeks before his death — then I left for Canada and heard of his death there. — His last words which could be understood were O(1) — may his Theorems live forever.

I believe that Turán's most important and most original work was the discovery and development of the so-called power sum method. He applied this method to different branches of analysis and number theory. My contribution to this theory was minor and others are much more competent to write about it than I. Turán himself considered this as his most important work. One evening I found him working and asked him "what do you prove and conjecture". He was working on his book and answered "I am building my pyramid".

A particular strength of Turán was that he often found a question of a new type in a field which was far from his own. He generally solved a special case "just for orientation" as he was fond of saying and then he returned "home", i.e. to analytic number theory. This happened e.g. in the theory of extremal graphs now a flourishing subject (in this case though Turán occasionally returned and made some further important contributions) and in the theory of relations in set theory and also in probabilistic number theory. No doubt his important contributions in statistical group theory and in the theory of interpolation will live for the foreseeable future.

To end I can only quote from Hilbert's famous obituary of Minkowski, I have to be grateful to fate that it gave me such a friend and co-worker and that I could have him for such a long time.

Paul Erdős