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Stability of continuously differentiable interval maps under C^1-perturbations

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Abstract. Consider a C^1-map $T : [0,1] \to [0,1]$ satisfying that $\{c \in (0,1) : T'c = 0\}$ is finite. Fix an $N \in \mathbb{N}$ with $N \geq \operatorname{card}\bigl(\{c \in (0,1) : T'c = 0\}\bigr) + 1$. Denote the family of all C^1-maps $S : [0,1] \to [0,1]$, which are piecewise monotonic with at most N intervals of monotonicity by $\mathcal{M}_N$. Obviously the conditions on T imply that $T \in \mathcal{M}_N$. The stability of T under small perturbations in $\mathcal{M}_N$ endowed with the C^1-topology (this means with respect to the norm $\|S\| := \max_{x\in[0,1]} |Tx| + \max_{x\in[0,1]} |T'x|$) will be investigated. Note that it is essential to assume that the number of intervals of monotonicity of the perturbation is bounded by the previously fixed number N, but one can choose this number arbitrarily large.
For every continuous function $f : [0,1] \to \mathbb{R}$ the topological pressure is upper semi-continuous at T, this means $\limsup_{\widetilde{T}\to T} p(\widetilde{T}, f) \leq p(T, f)$. If f satisfies that $p(T, f) > \lim_{n\to\infty} \frac{1}{n} \max_{x\in[0,1]} \sum_{j=0}^{n-1} f(T^j x)$ (this holds, for example, if $p(T, f) > \max_{x\in[0,1]} f(x)$), then the topological pressure is continuous at T. These results imply that the topological entropy is continuous. An example shows that the topological pressure is not lower semi-continuous at T in general. Finally, assume that $h_{\mathrm{top}}(T) > 0$ and T has a unique measure μ of maximal entropy. Then there exists an open neighbourhood U of T in $\mathcal{M}_N$ (with respect to the C^1-topology), such that every $\widetilde{T} \in U$ has a unique measure $\mu_{\widetilde{T}}$ of maximal entropy, and $\lim_{\widetilde{T}\to T} \mu_{\widetilde{T}} = \mu$ in the weak star-topology.

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